# GENERATION OF FREQUENCY MODULATED SIGNALS AT MICROWAVES

RALPH BENNETT NEAL

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Ralph B. Neal

# GENERATION OF FREQUENCY MODULATED SIGNALS AT MICROWAVES

by

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Captain, United States Marine Corps

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## PREFACE

Present trends indicate that the microwave frequency range will be used extensively for communication systems. Few such systems except those of the radio relay type are employed today, but as crowding continues in the UHF region, and the desire for greater security increases, it is believed that much of the traffic destined for nearby locations will be shifted to "point to point" systems in the microwave range. Further, it is believed that the frequency modulation technique allows utilization of most of the advantages found in this range without introducing undue complexity to the system.

The one operation for a communication system at microwave frequencies that is different from those encountered either in low frequency communication systems or in radar systems is the frequency modulated signal generation. Because of this difference, consideration of such signal generation is of interest. Therefore this paper presents descriptions and analyses of the more promising devices and methods for producing frequency modulated signals at microwave frequencies. By comparing the results shown herein, the device or method most suitable for a given system may be selected. Where applicable, specific information may be substituted into the equations given for frequency, modulation sensitivity, and distortion to obtain an estimate of these factors for a particular situation. One exception to the above statement must be made. The reactance tube method is not considered a practical or promising system at the present stage of development. Its inclusion is for comparison purposes with the other systems in order to form a bridge between the frequency modulation methods

employed at low frequencies and those encountered at microwaves. For example, by such comparison, it may be seen that the microwave devices tend to introduce only a small percentage of the distortion that is inherent in the reactance tube modulator.

Appreciation is expressed to Dr. William Beaver and other officers and employees of Varian Associates for the help and information given for the klystron and backward wave oscillator chapters, and for the opportunity to make the experimental measurements on the VA-180 and VA-181 backward wave oscillator tubes. Appreciation is further expressed to Professor Gene Cooper and Professor Carl Menneken for their aid and constructive criticism during the preparation of this paper.

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TABLE OF SYMBOLS AND ABBREVIATIONS

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| | |
|---|---|
| $F_x, F_y, F_z$ | Components of Force |
| m- | Mass of Electron ($9.103 \times 10^{-31}$kg) |
| e | Charge of Electron ($-1.602 \times 10^{-19}$ coulomb) |
| H | Magnetic Field Strength |
| $\omega_c$ | Cyclotron Frequency $= \frac{He}{m}$ |
| $\omega$ | The Operating Frequency |
| $E_o$ | Peak Time Varying Field between Plates |
| $v_x$ | Electron Velocity Component |
| $I_o$ | D.C. Beam Current |
| i | Instantaneous Current to Lower Plate |
| $v_o$ | Electron Velocity in z Direction (Constant) |
| d | Distance between Plates |
| l | Length of Plates |
| $V_{ac}$ | Peak Voltage between Plates $E_o d$ |
| $Y_e$ | Electronic Admittance Due to Modulating Beam |
| $G_e$ | Real or Conductance Part of $Y_e$ |
| $B_e$ | Imaginary or Susceptance Part of $Y_e$ |
| $V_o$ | Cathode to Parallel Plates D.C. Potential |
| $\tau$ | Time for Electron to Travel through Length of Plates $\left(\frac{l}{v_o}\right)$ |
| $\theta$ | Total Phase Difference in Time Required for Electron Transit $[(\omega_c - \omega)\tau]$ |
| $B_c$ | Susceptance of Loaded Cavity |
| $C_o$ | Equivalent Cavity Capacitance |

| | |
|---|---|
| $L_0$ | Equivalent Cavity Inductance |
| gm | Transconductance of Modulating Beam Structure |
| $e_g$ | Grid Voltage Applied to Modulator |
| Chapter III | |
| $Y_e$ | Beam Admittance |
| Yc | Cavity Admittance (Loaded) |
| $G_o$ | D.C. Beam Conductance $I_o/V_o$ |
| A | Beam Coupling Coefficient |
| $\tau_o$ | D.C. Transit Angle $\left(\frac{4\pi f d \sqrt{2\frac{m}{e}V_o}}{V_R+V_o}\right)$ |
| k | Bunching Parameter $\left(\frac{A\alpha\tau_o}{2}\right)$ |
| $\alpha$ | Excitation-Voltage Ratio $V_1/V_o$ |
| $V_o$ | D.C. Beam Voltage |
| $V_1$ | Peak r.f. Voltage between Grids |
| $J_1(k)$ | Bessel Function, First Order, First Kind with Argument k |
| $G_c$ | Cavity Conductance (Loaded) Referred to Grids |
| Q | Cavity Figure of Merit |
| f, $\omega$ | Operating Frequency |
| $f_o, \omega_o$ | Natural Frequency of Loaded Cavity |
| d | Distance from Grids to Repeller |
| m | Mass of Electron ($9.103 \times 10^{-31}$kg) |
| e | Charge of Electron ($1.602 \times 10^{-19}$ coulomb) |
| $V_R$ | Repeller or Reflector Potential |
| S | Modulation Sensitivity, $\frac{df}{dV_R}$ |
| $V_R'$ | D.C. Repeller Voltage |
| v | A.C. Repeller Voltage ($v = v_m \sin\omega_m t$) |
| vm | Peak A.C. Repeller Voltage |

| | |
|---|---|
| $\omega_m$ | Modulating Frequency |
| $G_L$ | Load Conductance Referred to Cavity Grids |
| $G_R$ | Resonator or Cavity Conductance (Unloaded) |
| $P_{out}$ | Output Power |
| g | $\equiv \frac{-(G_L + G_R)}{G_0 A^2 T_0 \sin T_0}$ |

Chapter IV

| | |
|---|---|
| $\beta$ | First Backward Wave Propagation Constant |
| $\beta e$ | Electron Beam Propagation Constant |
| L | Length of r.f. Structure |
| $\beta o$ | Fundamental Propagation Constant |
| p | Periodicity of r.f. Structure |
| n | Number of Spatial Harmonic |
| f, $\omega$ | Operating Frequency |
| $v_z$ | Velocity in Z Direction |
| a | Pitch Radius of Helix |
| $v_h$ | Velocity along Wires of Helix |
| k' | Modified Propagation Constant of Wave through r.f. structure |
| $u_o$ | Average Velocity of Electron Beam |
| e | Charge of Electron ($1.602 \times 10^{-19}$ coulomb) |
| m | Mass of Electron ($9.103 \times 10^{-31}$ kg) |
| $V_o$ | Beam Potential |
| d' | Dielectric Loading Factor |
| c | Velocity of Light |
| $\epsilon'$ | Relative Dielectric Constant |
| S | Modulation Sensitivity |
| $V_o'$ | D.C. Component of Beam Potential |

| | |
|---|---|
| $v$ | A.C. Component of Beam Potential ($v = v_m \sin \omega_m t$) |
| $v_m$ | Peak of A.C. Component of Beam Potential |
| $\omega_m$ | Modulating Frequency |
| $D_2$ | Second Harmonic Distortion |
| Chapter V | |
| $I_c$ | Current through Series Circuit of R and C |
| $E_p$ | Voltage across Oscillator Tuned Circuit |
| R | Resistance in Grid Circuit of Reactance Tube |
| C | Capacitance from Grid to Plate of the Reactance Tube |
| $f, \omega$ | Operating Frequency |
| $E_g$ | r.f. Component of Reactance Tube Grid Voltage |
| $\mu$ | Amplification Factor of Reactance Tube |
| $r_p$ | Plate Resistance of Reactance Tube |
| $I_p$ | r.f. Component of Plate Current of Reactance Tube |
| $g_m$ | Transconductance of Reactance Tube |
| Y | Equivalent Admittance of Reactance Tube Circuit across Oscillator Tuned Circuit |
| $g_o$ | Transconductance Value at Intercept of Straight Line through Linear Portion of $g_m$-$E_c$ Curve and $g_m$ Axis |
| $E_c$ | Grid Voltage Value at Intercept between Straight Line Extension of Linear Part of $g_m$-$E_c$ Curve and Grid Voltage Axis |
| $e_c$ | Instantaneous Reactance Tube Grid Voltage |
| $L_o$ | Oscillator Tuned Circuit Inductance |
| $C_o$ | Fixed Capacitance Across Oscillator Tuned Circuit |
| S | Modulation Sensitivity $\frac{df}{de_c}$ |
| $E_{cc}$ | D.C. Reactance Tube Grid Bias |
| $v$ | A.C. Component of Reactance Tube Grid Voltage |

| | |
|---|---|
| $v_m$ | Peak Value of A.C. Component of Reactance Tube Grid Voltage |
| $\omega_m$ | Modulating Frequency |
| $D_2$ | Second Harmonic Distortion |
| $f_o$ | Frequency of Oscillator for Reactance Tube Grid Voltage of $E_{cc}$ only |

# CHAPTER I

## INTRODUCTION

The use of higher and higher frequencies for communication purposes has been a continuing process throughout the history of radio. Now, with the advent of microwave relay and other special short hop systems, it has arrived in the kilomegacycle region which heretofore was employed almost exclusively for radar, radar beacon, and experimental work. Just where the upper-frequency limit for practical communication systems lies is a question which has been the basis for considerable debate. Because of the adverse effect of certain types of weather at higher frequencies, the limit for systems of high reliability has sometimes been set at about 5 kilomegacycles. Other systems have been used in the 10 kilomegacycle range where the reliability requirements are not as great. From a military point of view security of transmission is of considerable importance. Here the failure of the higher frequencies to carry for long distances together with the very narrow antenna beamwidths that can be obtained with relatively small antennas can be extremely desireable. These characteristics may have particular importance for those many occasions where links a few feet to a few hundred feet long are needed, such as between lower echelon infantry units. Therefore it seems highly probable that there will be requirements in the communication field for any of the higher frequencies which can be generated coherently by simple and reliable devices.

In the microwave range, which we shall tacitly assume to include all signals above about one kilomegacycle (and presently extending

upward to more than 30 kilomegacycles), frecuency modulation appears to be one of the most satisfactory methods for impressing the intelligence on the carrier. This method allows the employment of greater bandwidths than are obtained with simple amplitude modulation, thus permitting the accompanying improvement in the signal-to-noise ratio. It does not, however, incur the additional complexity which pulse modulation systems introduce. Frequency modulation is the type of modulation used most extensively in present day eouipments.

This paper investigates some of the devices and methods for producing frequency modulated signals in the microwave region. Because of the large number of devices proposed or in existence, and especially because of the extensive number of differences in similar such devices, no attempt is made to survey the entire field. However, individual types of the devices which are presently most popular, or which appear to be particularly adapted for frequency modulation with considerable ease, but which have not yet been developed, are considered. One exception to this is the analysis of the reactance tube method which, at present, does not appear to be used or to have great possibilities. This analysis was included for comparison purposes only.

Specifically, the devices or methods herein considered are the frequency modulated magnetron, the klystron, the backward wave oscillator, and the reactance tube modulator. The frequency dependence upon the particular variable used for frequency modulation is shown in all cases, and the modulation sensitivity and second harmonic distortion are shown for the last two items listed. In addition, an attempt is made to explain the mechanics of operation sufficiently to provide an approximate model.

Where information was available in the literature or could be obtained on tubes physically available, experimental information is presented for comparison with the theoretical results. Finally, the capabilities of experimental models, or actual commercial models in the case of the klystron, are presented to show what has been done with the particular device or method being considered. In the case of the backward wave oscillator, some further information on its basic construction and operation is given because of its relative newness and because of the accompanying general lack of information on the tube at present.

In this paper, the rest of the communication system is not considered. The expressions given for second harmonic distortion are for the modulation system or method only and assume a perfect discriminator.

## CHAPTER II

## THE FREQUENCY MODULATED MAGNETRON

Before going into a consideration of some of the cw magnetrons capable of being frequency modulated, an explanation of the general principle utilized to effect this modulation is in order. Effectively, this principle results in the change in admittance of a cavity; such change occurring when an electron beam is passed through the slots of a cavity parallel to a static magnetic field, and at right angles to a varying electric field, such as the field at the output frequency that exists across the slots of the cavities of a magnetron. Note that the relationship between the static magnetic field and the time varying electric field in a conventional magnetron satisfies the requirement so that it is only necessary to introduce the electron beam into the cavities parallel to the magnetic field. In that the amount of susceptance change in a cavity under the above conditions is approximately proportional to the beam current, this susceptance may be made to follow a modulating signal by density modulating the beam with this signal. Herein lies the ability to obtain frequency modulation of a magnetron in that the effective admittance of the cavity plays an important part in determining the output frequency.

To investigate this effect, let us consider the arrangement shown in Fig. 1, in which we have two parallel plates separated by a distance d in the x direction, of length l in the z direction, and infinite in the y direction. Time varying currents to the lower plate will be $i = I\epsilon^{j\omega t}$, and the static magnetic field in the z direction will have

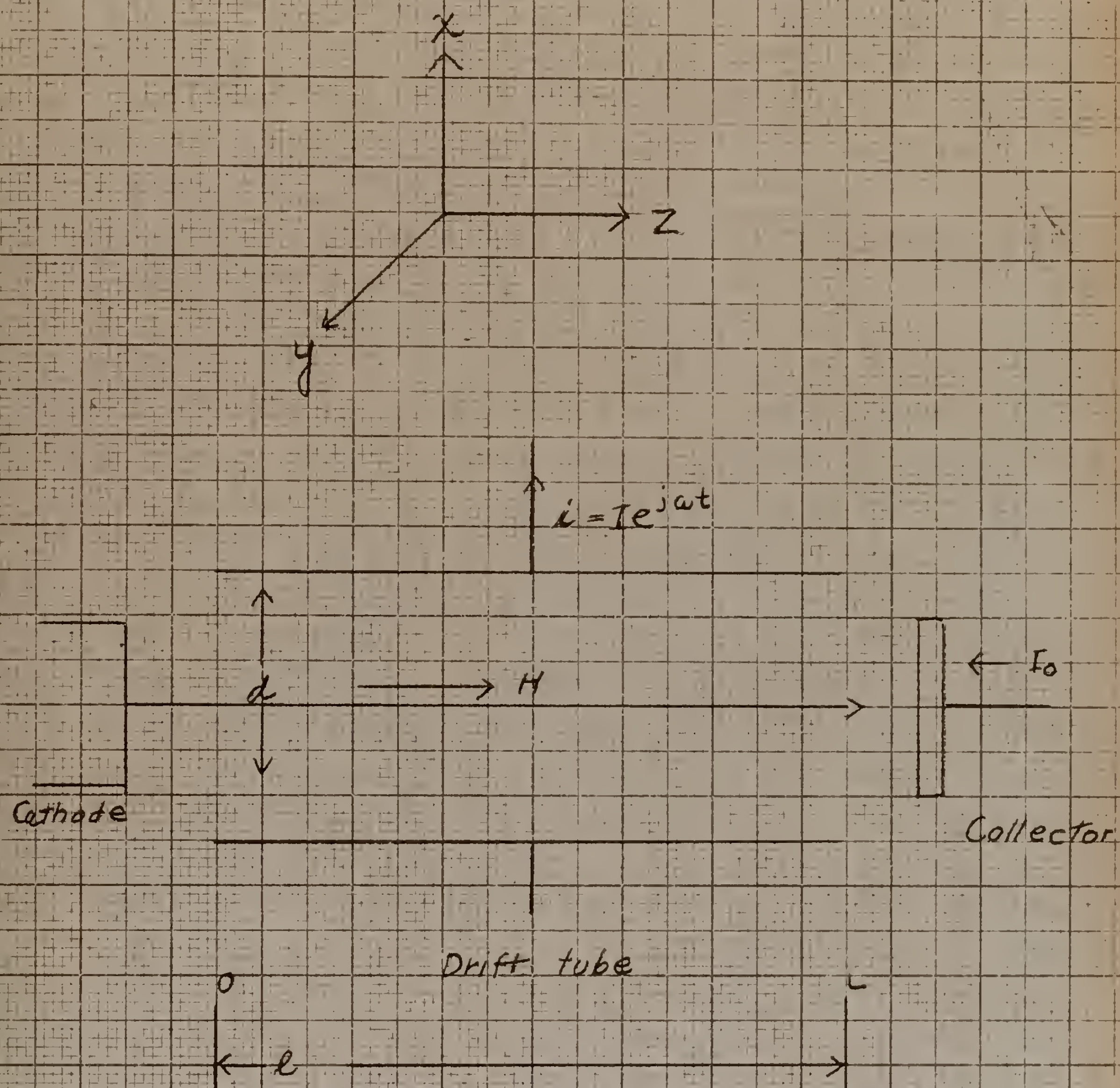



Fig 1

Sketch of Spiral Beam and Parallel Plates for Impedance Analysis

a magnitude H. Then following generally the approach of Smith and Shulman,[12] and neglecting fringing field effects, the electron ballistic equations are

$$F_x = m\ddot{x} = E_x e + He\dot{y}$$
$$F_y = m\ddot{y} = -He\dot{x}$$
$$F_z = m\ddot{z} = 0$$

also at $t = t_0$, $x = z = \dot{x} \equiv v_x = \dot{y} = 0$ and $\dot{z} = v_0$

Then $\dot{y} = \frac{-He}{m}x$ which when substituted into the $F_x$ equation gives

$$\ddot{x} + \left(\frac{He}{m}\right)^2 x = eE_x = \frac{eE_0}{m}\epsilon^{j\omega t}$$

Now let $\frac{He}{m} = \omega_c$, which will be recognized as the cyclotron frequency. The general solution to the complete equation above will then be

$$x = C_1\epsilon^{j\omega_c t} + C_2\epsilon^{-j\omega_c t} + \frac{eE_0}{m(\omega_c^2-\omega^2)}\epsilon^{j\omega t}$$

in which the integration constants $C_1$ and $C_2$ must be determined from the initial conditions and are found to be

$$C_1 = \frac{-eE_0}{2\omega_c m(\omega_c-\omega)}\epsilon^{j(\omega-\omega_c)t_0}$$

and

$$C_2 = \frac{-eE_0}{2\omega_c m(\omega_c+\omega)}\epsilon^{j(\omega+\omega_c)t_0}$$

which upon substitution in the above solution and the differentiating gives

$$v_x = \dot{x} = \frac{jeE_0}{m(\omega^2-\omega_c^2)}\left[\frac{\omega_c(\omega-\omega_c)\epsilon^{-j(\omega_c+\omega)(t-t_0)}}{2\omega_c} + \frac{\omega_c(\omega+\omega_c)\epsilon^{j(\omega_c-\omega)(t-t_0)}}{2\omega_c} + \omega\right]\epsilon^{j\omega t}$$

For our range of interest $\omega_c \cong \omega$, therefore the first term will be quite small compared to $\omega$, and further it is nearly twice the frequency of operation. Therefore it will be neglected, giving

$$v_x \cong \frac{jeE_0}{m(\omega^2-\omega_c^2)}\left[\frac{\omega_c(\omega+\omega_c)}{2\omega_c}\epsilon^{j(\omega_c-\omega)(t-t_0)} + \omega\right]\epsilon^{j\omega t}$$

or upon further taking $\frac{\omega}{\omega+\omega_c} \cong \frac{1}{2}$

$$v_x \cong \frac{jeE_0}{2m(\omega-\omega_c)}\left[\epsilon^{j(\omega_c-\omega)(t-t_0)}+1\right]\epsilon^{j\omega t}$$

The incremental current in the plates due to the incremental charge, dq, moving in the x direction is,[13] neglecting the time varying term for the present,

$$di = \frac{dq}{d}v_x = \frac{I_0 v_x}{v_0 d}dz$$

$$= \frac{jE_0 e I_0}{2m(\omega-\omega_c)v_0 d}\left[\epsilon^{j(\omega_c-\omega)\frac{z}{v_0}}+1\right]dz$$

where $t = t_0 + \frac{z}{v_0}$

To get the total current, this expression is integrated from 0 to L as

$$I = \frac{jeE_0 I_0}{2m(\omega-\omega_c)v_0 d}\int_0^L\left(\epsilon^{j(\omega_c-\omega)\frac{z}{v_0}}+1\right)dz$$

or

$$I = \frac{jeE_0 I_0}{2m(\omega-\omega_c)v_0 d}\left[\frac{jv_0}{\omega_c-\omega}\left(1-\epsilon^{j(\omega_c-\omega)\frac{L}{v_0}}\right)+L\right]$$

which if $\tau = \frac{L}{v_0}$ and $\theta = (\omega_c-\omega)\tau$

becomes

$$I = \frac{eE_0 I_0}{2md}\tau^2\left[\frac{1-\cos\theta}{\theta^2}+j\frac{\theta-\sin\theta}{\theta^2}\right].$$

Now if

$V_{ac} = E_0 d$ and with $Y_e = \frac{I}{V_{ac}}$

$$Y_e = \frac{eI_0}{2md^2}\tau^2\left[\frac{1-\cos\theta}{\theta^2}+j\frac{\theta-\sin\theta}{\theta^2}\right]$$

or further with $V_0 = \frac{1}{2}\frac{m}{e}v_0^2 = \frac{1}{2}\frac{m}{e}\frac{L^2}{\tau^2}$

becomes

$$Y_e = G_e + jB_e = \frac{L^2 I_0}{4d^2 V_0}\left[\frac{1-\cos\theta}{\theta^2}+j\frac{\theta-\sin\theta}{\theta^2}\right].$$

This equation, then, represents the admittance of the beam referred to the parallel plates.

A plot of the factors within the parentheses as a function of $\theta$ is shown in Fig. 2. From these curves it is readily seen that the electronic admittance has both real and imaginary components for any discrete length of plates up to $\theta$ equal to $2\pi$ for which length the real part becomes zero. It may further be seen that the imaginary part of this admittance, which is the effective part in changing the frequency of the magnetron, is reduced to only one-half its maximum value. The maximum value occurs for a value of $\theta$ equal to $\pi$. This larger imaginary value would be desirable for producing a frequency change, however the real component of the electronic admittance is still 40% of its maximum value for this length of plates, and the accompanying amplitude modulation caused by this electronic conductance is generally undesirable so that the length corresponding to $\theta$ equal to $2\pi$ is of greater interest, and is the design figure in practice.

At this time, after considering the first order mathematical determination of the character of the electronic admittance due to the beam, it is of interest to consider the mechanics of the interaction between the fields and the beam which creates this admittance. An electron entering the fields between the plates at "a" in Fig. 3 at first has only velocity in the $z$ direction. However, as it comes under the influence of the time varying electric field, it is accelerated toward whichever of the plates is positive at the time. For our consideration, say that the upper plate of Fig. 3 is positive. The electron then moves toward this positive plate. As it gains an $x$ component of velocity, it

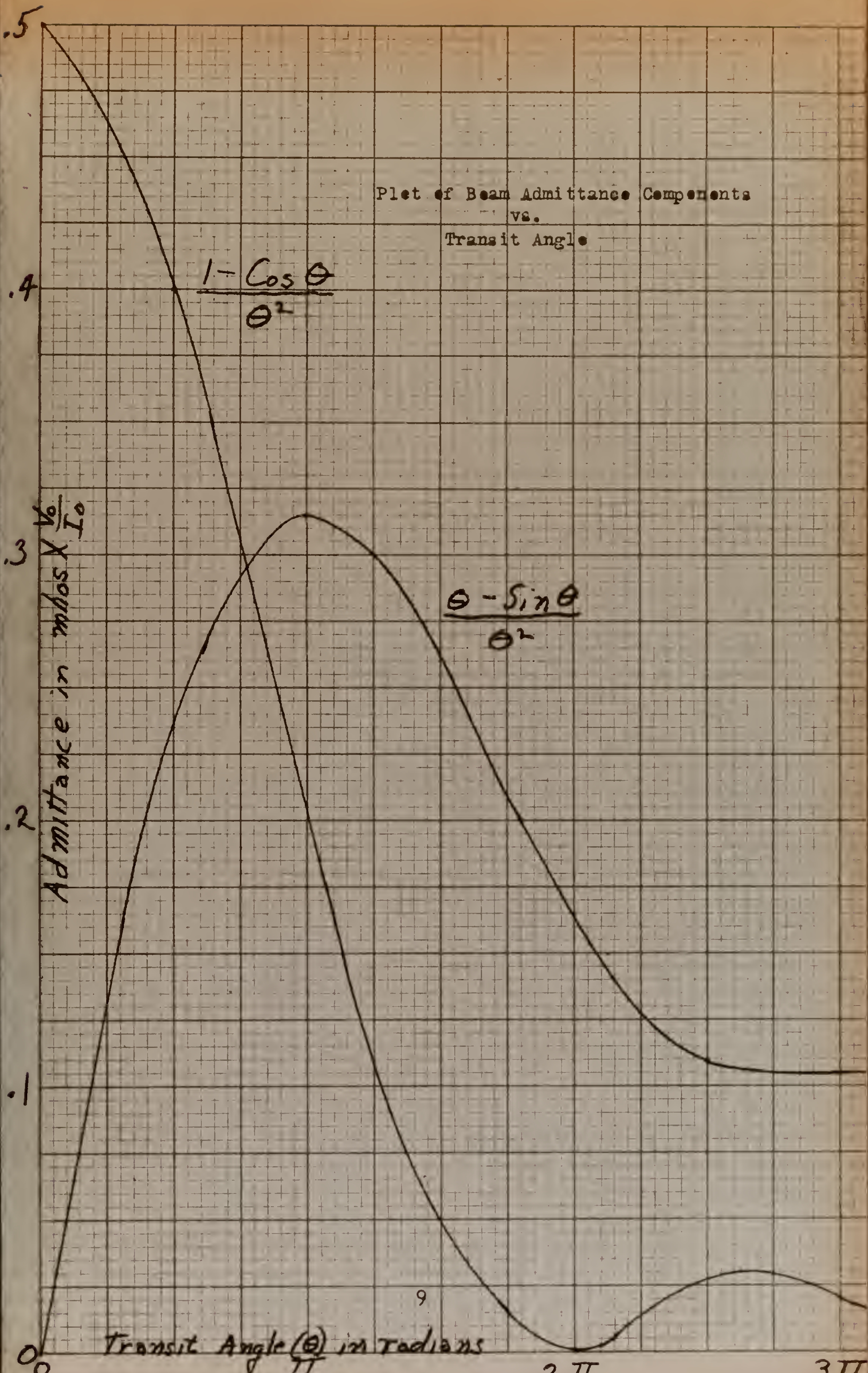
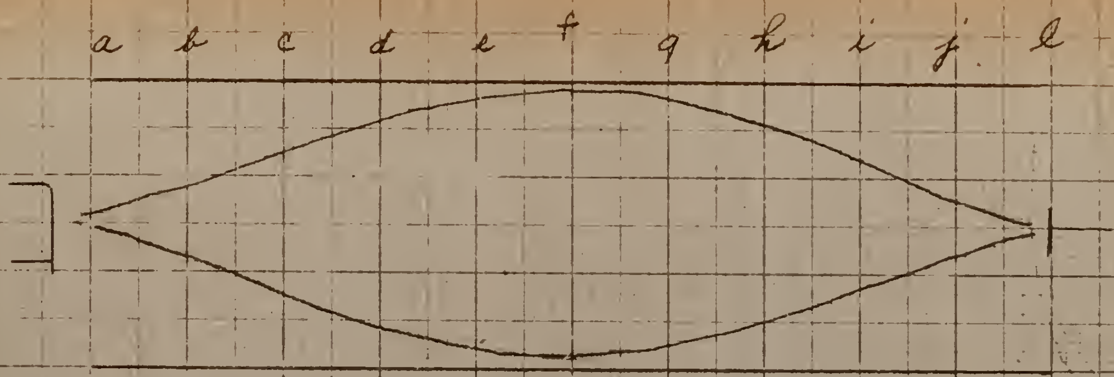
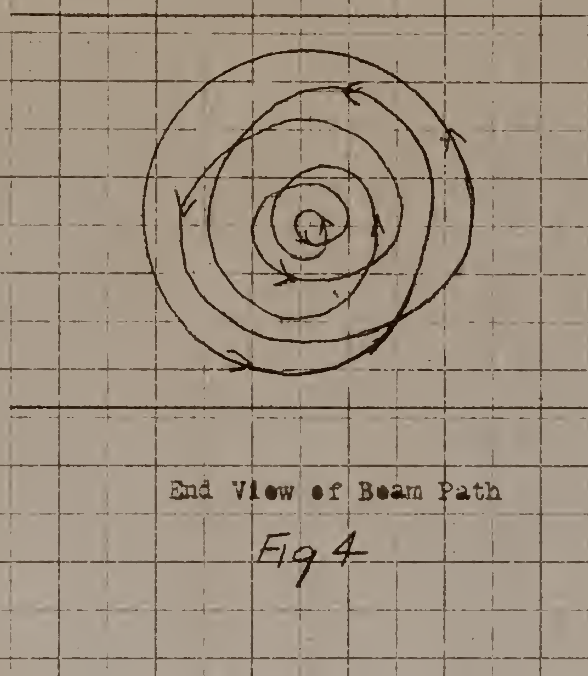

Plot of Beam Admittance Components
vs.
Transit Angle
$\frac{1-\cos\theta}{\theta^2}$
$\frac{\theta-\sin\theta}{\theta^2}$
Admittance in mhos X $\frac{V_0}{I_0}$
.5
.4
.3
.2
.1
0
0
$\pi$
$2\pi$
$3\pi$
Transit Angle ($\theta$) in radians

Fig 2



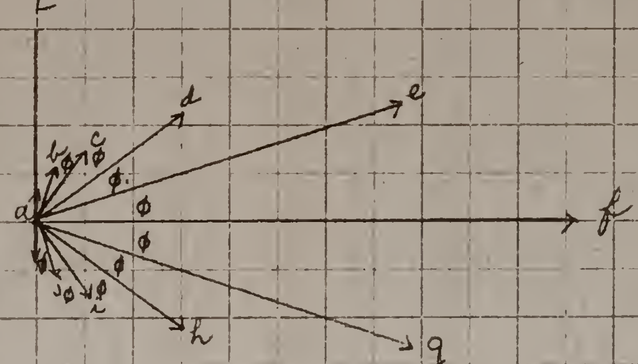

Side View of Beam Path

Fig 3



End View of Beam Path

Fig 4




Vector Diagram of Current Increments Along Beam

Fig 5

interacts with the magnetic field which gives it a $y$ component of velocity. In fact, the electron will tend to describe a circular path in the x-y plane at a frequency determined solely by the intensity of the magnetic field, namely $\omega_c = \frac{He}{m}$. Now if $\omega_c$ is less than the frequency, $\omega$, of the time varying field between the plates, the electron will lag farther and farther behind the field. Until the electron is 90 degrees behind the field in phase, it will receive energy from the field, and its path will be that of a helix of increasing radius. As the electron lags behind the field by more than 90 degrees but by less than 180 degrees, it will be working into a retarding field, returning energy to the field, and, consequently, describing a helix of decreasing radius. When the length of the plates, and other conditions, are such that $\theta$ is equal to $2\pi$, the electron will have returned all of the energy that was taken from the field. The path of an electron under these conditions is depicted in Fig. 3 and Fig. 4 except that the number of revolutions in the spiral would in general be many times that shown in Fig. 4.

Now consider the induced current in the plates due to the motion of electrons at several equally spaced points along the length of the plates. This induced current will be in phase with the voltage across the plates at "a" in Fig. 3 but of a relatively small amplitude. The current will lag behind the voltage by equal angles, $\phi$, for each successive position down the plates. The current will also increase in magnitude as the helix radius increases until, at point "f" in Fig. 3 where $\theta$ is equal to $\pi$, the magnitude of the current will be maximum and the phase will be 90 degrees behind the field. The phase will continue to lag and the amplitude of the current will decrease until at L, where $\theta$ is equal to $2\pi$,

the current is 180 degrees out of phase with the voltage, and of zero amplitude. Fig. 5 represents these currents and shows that all will have a component 90 degrees out of phase with the voltage which will be additive, whereas the in-phase components will sum to zero. The total current due to the summation of all such electron currents over the length of the plates will be 90 degrees out of phase with the voltage, thus acting as a pure reactance in parallel with the plates. This is true, of course, only for conditions such that $\theta$ is equal to $2\pi$, or integer multiples thereof. The higher values of $\theta$ are of less interest, however, because the magnitude of the effect becomes less as shown in Fig. 2. This same approach is applicable also for $\omega_c$ greater than $\omega$ except that in this case, the currents due to the electron motion will lead the voltage across the plates. For conditions such that $\theta$ is not equal to an integer times $2\pi$, the in-phase component of the currents will not sum to zero, and the admittance will be complex.

Now that an electronic admittance for the modulating electron beam has been calculated, and a model of the action producing this admittance has been considered, it is of interest next to determine the effect of this admittance in changing the basic frequency of the magnetron. Even though circuit elements are distributed rather than lumped insofar as a magnetron cavity is concerned, for our purposes we may consider an equivalent inductance and capacitance based upon equivalence of stored energy in the magnetic and electric fields of the cavity, and the equivalent circuit elements with the same peak voltage across the equivalent capacitance as exists at the gap of the cavity, and the same current in the equivalent inductance that flows to and from the gap. These equivalent

circuit elements will be $C_o$ and $L_o$. Further they will include any effects due to loads coupled into the cavity, and any effects due to the imaginary component of the electronic admittance due to the power generating portion of the magnetron.

Now for the operating magnetron, it will be true that

$$B_e + B_c = 0$$

where

$$B_e = \frac{L^2 I_o}{4d^2 V_o} \frac{\theta - \sin\theta}{\theta^2}$$

is the electronic susceptance due to the modulating beam as previously shown, and

$$B_c = \omega C_o - \frac{1}{\omega L_o} = \frac{C_o}{\omega}\left(\omega^2 - \omega_o^2\right)$$

is the circuit susceptance and $\omega_o = \frac{1}{\sqrt{L_o C_o}}$ is the frequency of the magnetron with the modulating beam absent.

Now let the operating frequency $\omega = \omega_o + \Delta\omega$

where $\Delta\omega << \omega_o$.

$$B_e + B_c = \frac{L^2 I_o}{4d^2 V_o} \frac{\theta - \sin\theta}{\theta^2} + \frac{C_o\left[(\omega_o + \Delta\omega)^2 - \omega_o^2\right]}{\omega_o + \Delta\omega} = 0$$

which because $\frac{\omega_o}{\omega_o + \Delta\omega} \cong 1$ becomes

$$\frac{L^2 I_o}{4d^2 V_o} \frac{\theta - \sin\theta}{\theta^2} = -2C_o\Delta\omega$$

or

$$\Delta\omega = \frac{-L^2 I_o}{8C_o d^2 V_o} \frac{\theta - \sin\theta}{\theta^2} .$$

Again we will be mostly concerned with the case where $G_e$ becomes 0, or where $\theta = 2\pi$.

Then

$$\Delta\omega(2\pi) = \frac{-L^2 I_o}{8d^2 V_o C_o} \cdot \frac{1}{2\pi} = \frac{-L^2 I_o}{16\pi d^2 V_o C_o} .$$

Thus to a first approximation, the change in frequency is proportional

to the beam current, $I_o$. $I_o$ will vary with the voltage applied to the grid of the modulating beam gun in a manner akin to that in which the plate current of a low frequency vacum tube varies with grid voltage, namely

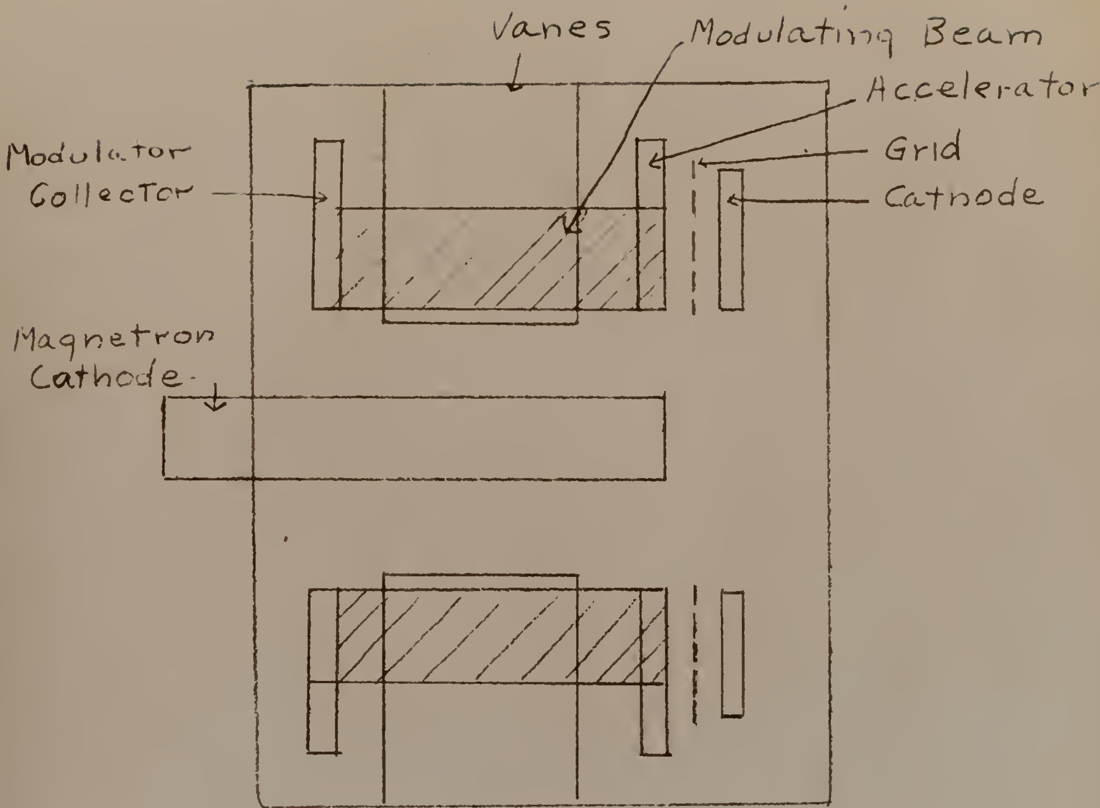
$$I_o = -g_m E_g \, .$$

Thus

$$\Delta\omega(2\pi) = \frac{L^2 g_m E_g}{16\pi d^2 V_o C_o}$$

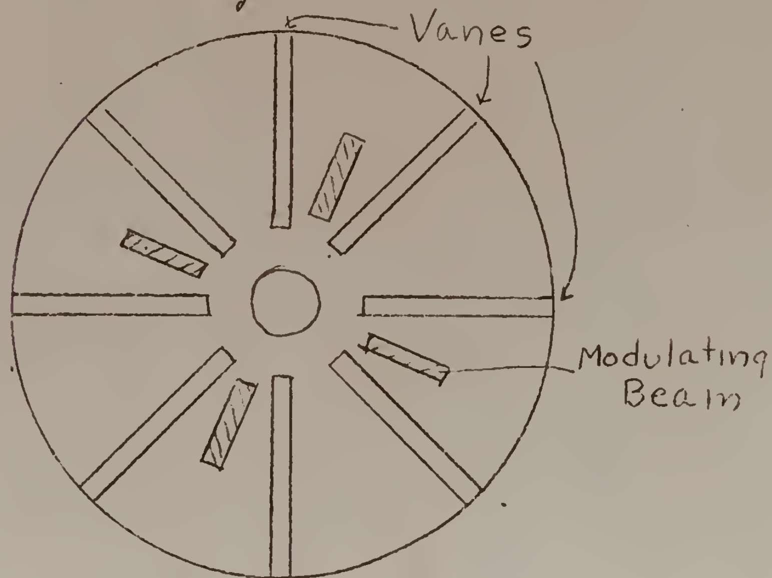
Having covered the first order analysis of frequency modulation by means of an auxiliary electron beam, it is now of interest to consider some of the practical applications of this principle as employed in FM magnetrons reported in the literature. Although the configuration of magnetron cavities departs from the ideal assumptions of infinitely wide parallel plates and no fringing fields used in the foregoing analysis, the results obtained therefrom will give approximate results for the conditions found in magnetrons, expecially in multivane magnetrons. The basic arrangement employed in these magnetrons is shown in Figs. 6 and 7. The range of control obtainable may be increased by increasing the number of modulation beams used, the maximum number of such beams being equal to the number of cavities.

One of the first such magnetrons reported upon in the literature is a CW magnetron in the 4 kmc, region built for use in experimental work on FM radar.[9] The magnetron developed about 25 watts CW with an anode potential of about 850 volts and an efficiency of about 50%. By using the modulation beam, a frequency deviation of about 2.5 mcs was obtained with very little amplitude modulation, and a frequency deviation




Side View of Typical Magnetron with Modulating Beam

Fig 6






End View of Typical Magnetron with Modulating Beam

Fig 7

of about 4 mcs was obtained by allowing moderate amplitude modulation. This tube was a twelve cavity vane type magnetron with modulating beams in two cavities. It is interesting to note that the modulating guns in this case (the tube was a laboratory device, not a production item) were made following standard receiver tube techniques and employing modified 6AK5 pentode receiving tubes. This tube also employed a mechanical slug tuner to give it a center frequency of from about 4.1 kmcs to about 4.25 kmcs. Figs. 8 and 9 show the static experimental results reported for this type of tube. The beam current is controlled by varying the grid voltage. The relatively high degree of linearity of frequency change vs beam current is of special interest in these curves. The current maximum of about 22 ma shown on the curves is the beam current at which interception of the modulating beam by the cavity plates commences to occur due to the increased size of the beam and the increased space charge effect.

One of the later tubes reported on is described in the June 1952 RCA Review.[5] This tube also is a tunable tube, operating over a range of from 6575 mcs. to 6875 mcs. Frequency deviations up to 16 megacycles are obtainable at rates up to 5 mcs. with relatively low amplitude modulation. Power output is reported at about 10 watts with the efficiency running between 30 and 40 percent. The frequency generating portion of the tube is a 24 vane, double strapped, system operating normally with an anode potential of 550 volts. Unlike the previous tube, the modulating beams are not located within the magnetron cavities, but rather are built into the tuning cavity. The operation, however, would be basically the same as before with the change in the tuning

Frequency Shift vs. Beam Current for 4000 mcs FM Magnetron

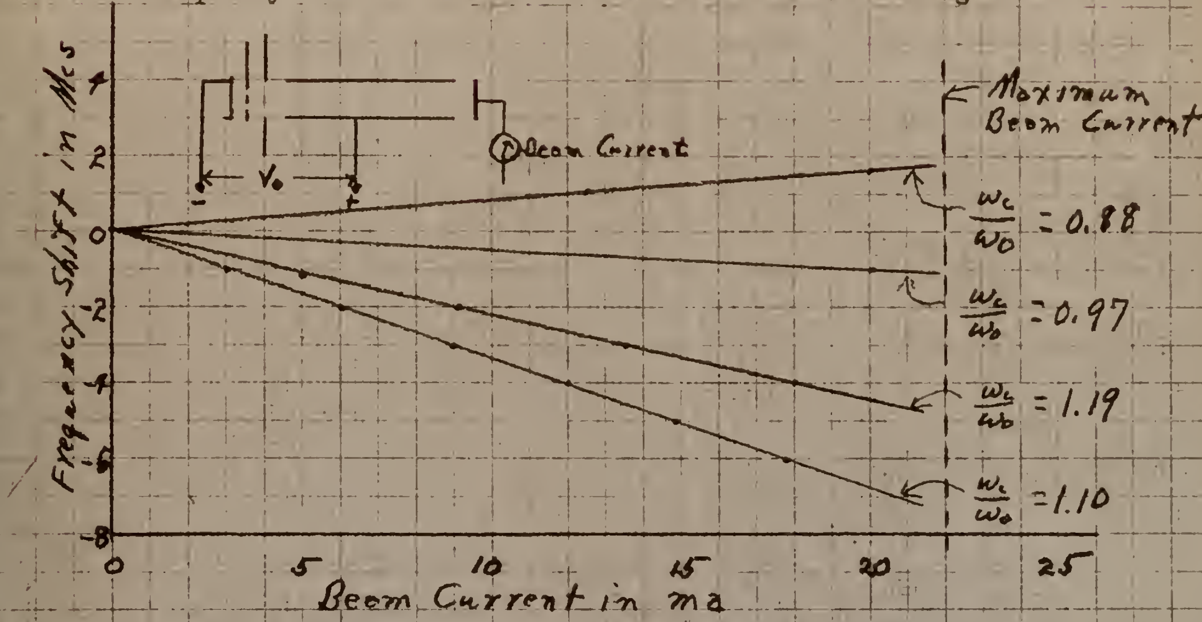



Fig 8

Power Output vs. Beam Current for 4000 mcs FM Magnetron

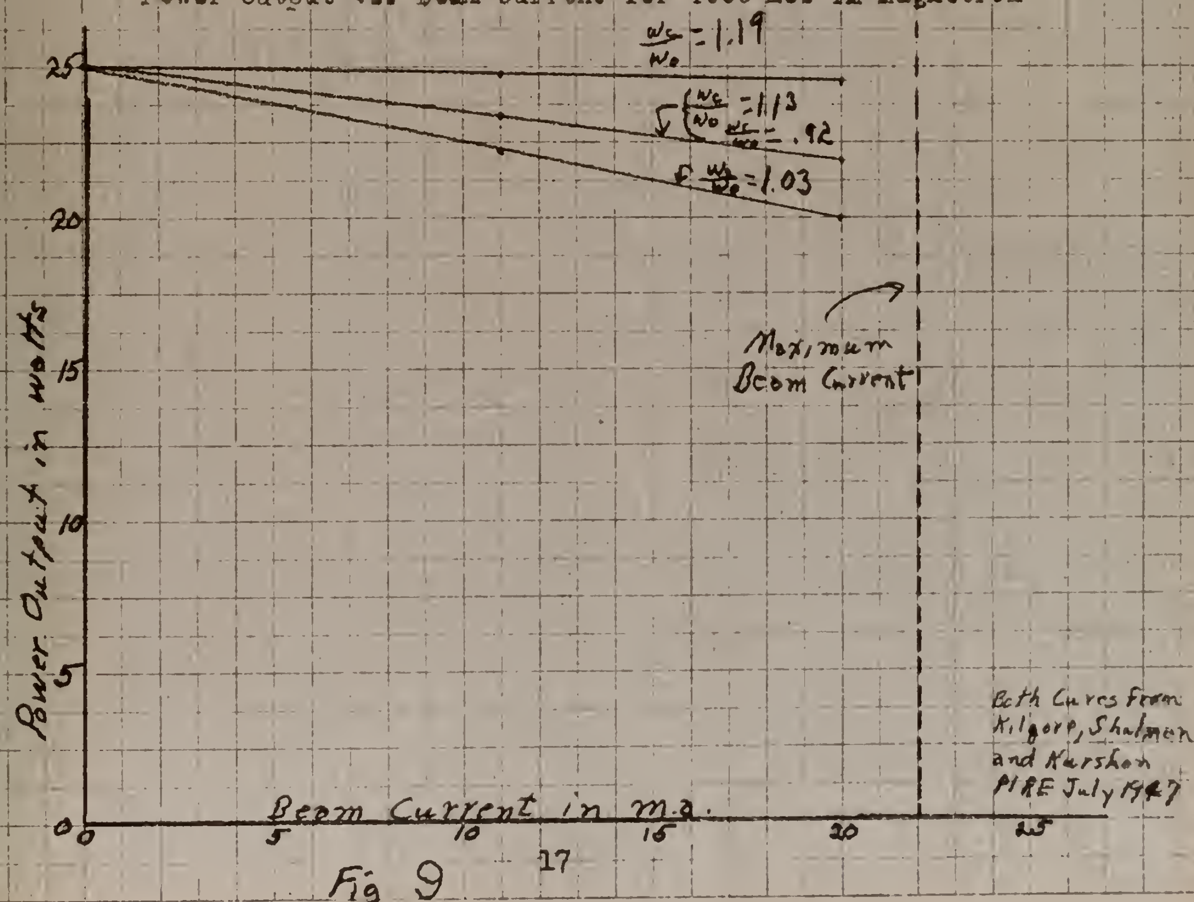



Fig 9

cavity admittance due to the beam being coupled into the main resonators through an iris to change the operating frequency. This change was undoubtedly dictated by the small size of the main resonators at this higher frequency.

Two guns are used in the modulation system of this tube. Fig. 10 shows the frequency deviation obtainable with each of the guns as a function of its control grid voltage. Either or both of the guns may be used. However, if both guns are used, they are run at half the current at which they would be run alone in order to give the same total current.

Because the tube can be designed for $|\theta|$ equal to $2\pi$ for only one frequency when a permanent magnet is used for the field, the FM modulation sensitivity and the amount of amplitude modulation will vary across the tuning range of the tube. Fig. 11 demonstrates this change. The FM will be noted to vary by about 25% across the tuning range while the amplitude modulation will become as high as 8 percent in the same range.

Although other magnetrons employing this principle of a "spiral beam" for obtaining frequency modulation have been constructed and reported on[2], it is believed that the previous examples demonstrate the results that can be obtained as well as the general ranges of control and currents or voltages necessary to obtain this control; therefore other such tubes will not be discussed here.

The spiral beam frequency modulated magnetron would appear to exhibit the following characteristics: it is capable of relatively high powers and efficiencies up to about 50%; frequency stability is rather good, and is not greatly effected by small supply voltage changes; and the tuning is of a restricted range, and the mechanism generally

Frequency Deviation vs. Grid Voltage for 7000 mcs
FM Magnetron

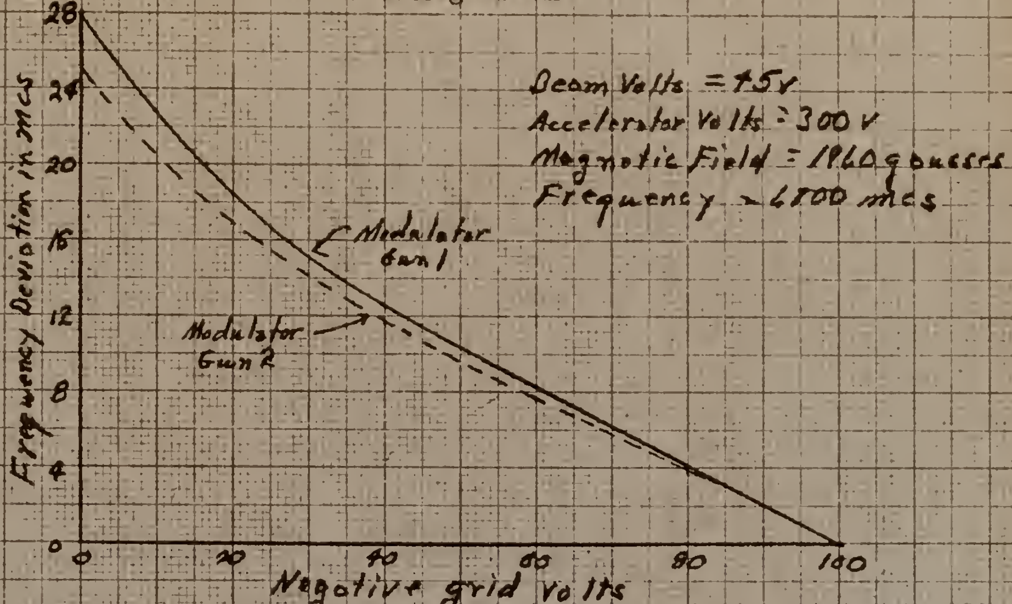



Fig 10

Both curves from Jenny
RCA Review, June 1952

Deviation and Amplitude Modulation
vs. Frequency
for 7000 mcs FM Magnetron

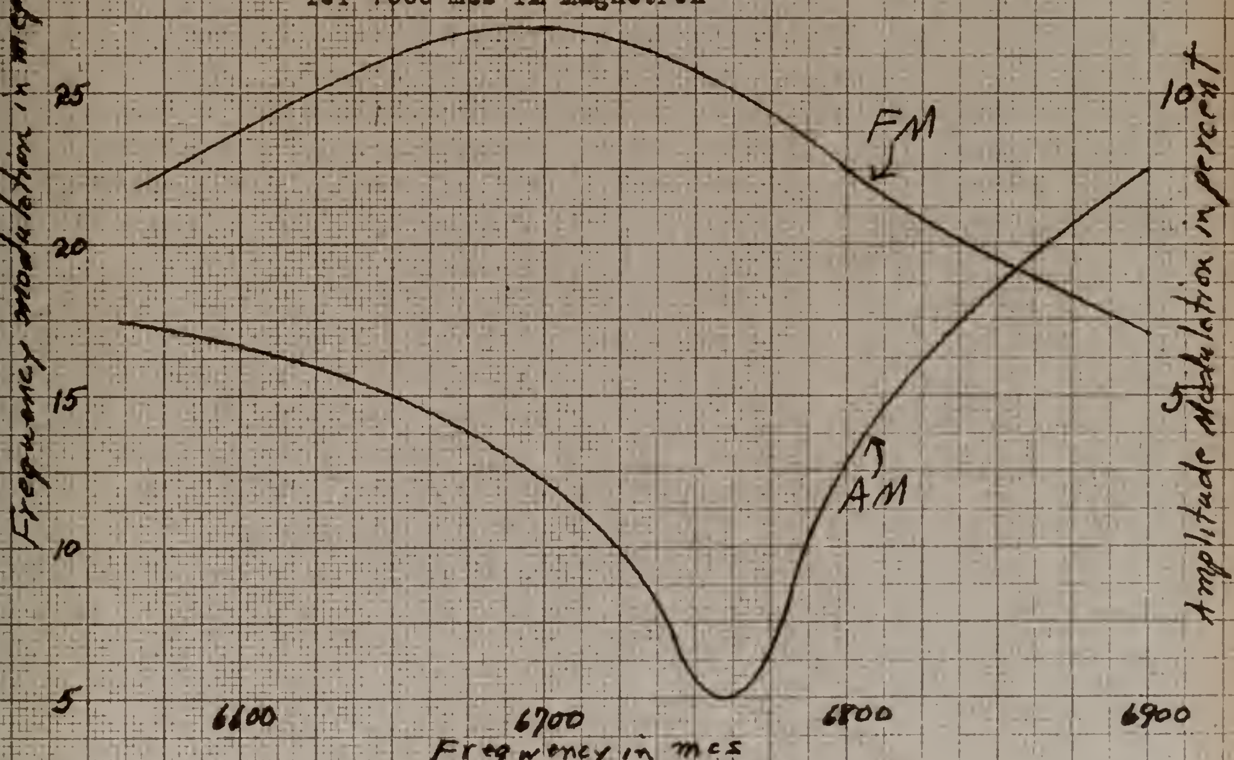



Fig 11

cumberson. Frequency modulation can be quite linear, but deviation does not tend to be high because it is limited by the combined limitations of the maximum beam current that can exist in the cavity space before space charge causes appreciable spreading of the beam, and the maximum beam spiral that can be used before interception with the plates occurs.

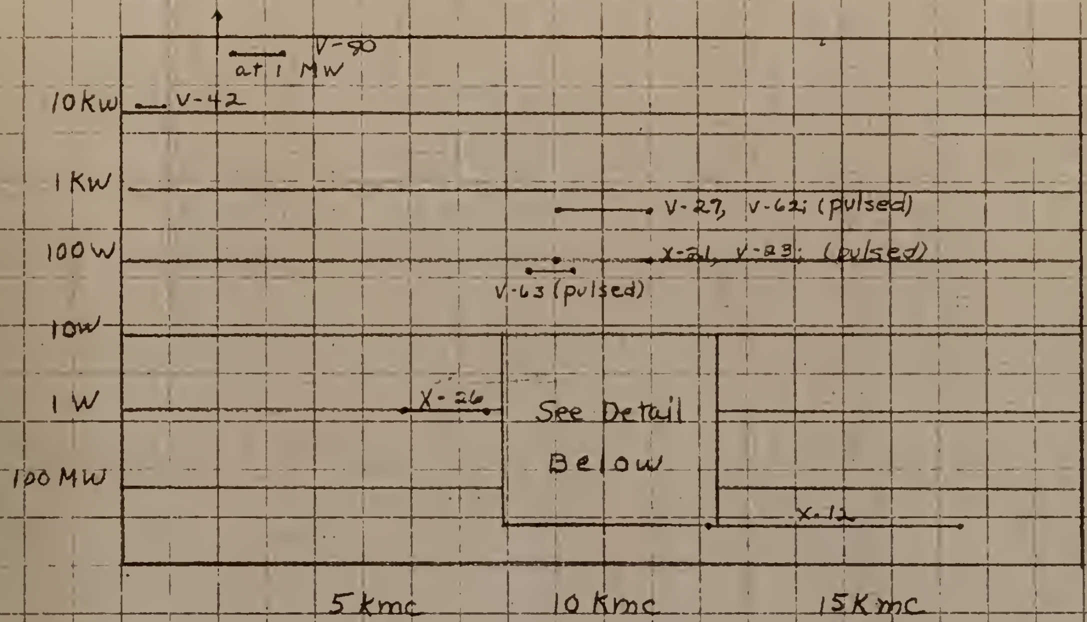
# CHAPTER III

## THE FREQUENCY MODULATION OF KLYSTRONS

The most satisfactory tube available for microwave communication work at the present time is the klystron. This type of tube, which has been used extensively in the past as a low power source for radar local oscillators, radar beacons, etc., has been extended by recent developments into the high power region. For example, one manufacturer, Varian Associates of Palo Alto and San Carlos, California, has klystrons in production ranging in frequency from 480 mcs to 17.5 kmcs, and capable of CW power outputs of up to 15 kw at the lower frequencies and up to about 10 watts at X band. The frequency ranges and power capabilities of some of these tubes are shown in the chart of Fig. 12. Also development of klystrons operating at S band with peak pulsed power outputs as high as 30 megawatts has been reported.[1]

Admittedly the higher powered klystrons are of the multicavity amplifier type; however, reflex oscillators with power outputs of a watt or more are also available. Further, the lower powered oscillators may be used as sources to drive the amplifiers provided that any frequency deviation of the oscillator is kept within the band pass of the amplifier.

The reflex oscillator of the klystron group readily lends itself to frequency modulation, and is, therefore, of greatest interest here in this discussion. This tube is quite sensitive to variations in supply voltages, both resonator-cathode potential and repeller-cathode potential. It is this sensitivity, which has required the use of well regulated power supplies and automatic frequency control circuits in local




Power Output and Frequency of Commercially Available Varian Klystrons

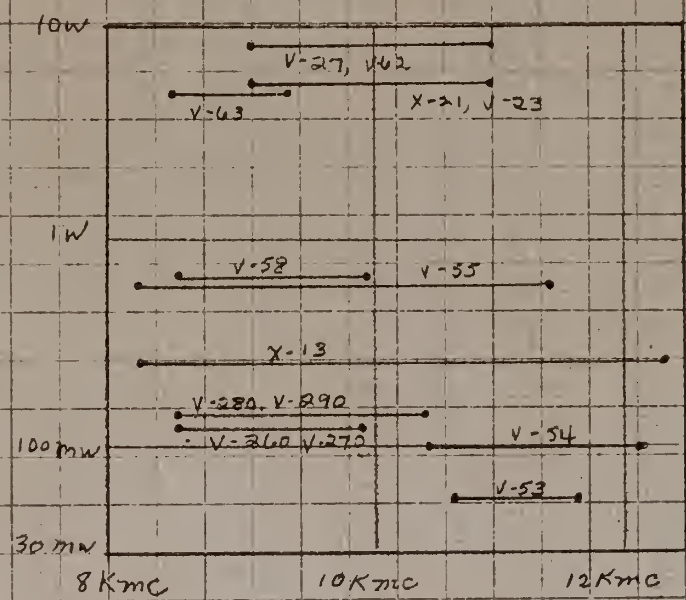


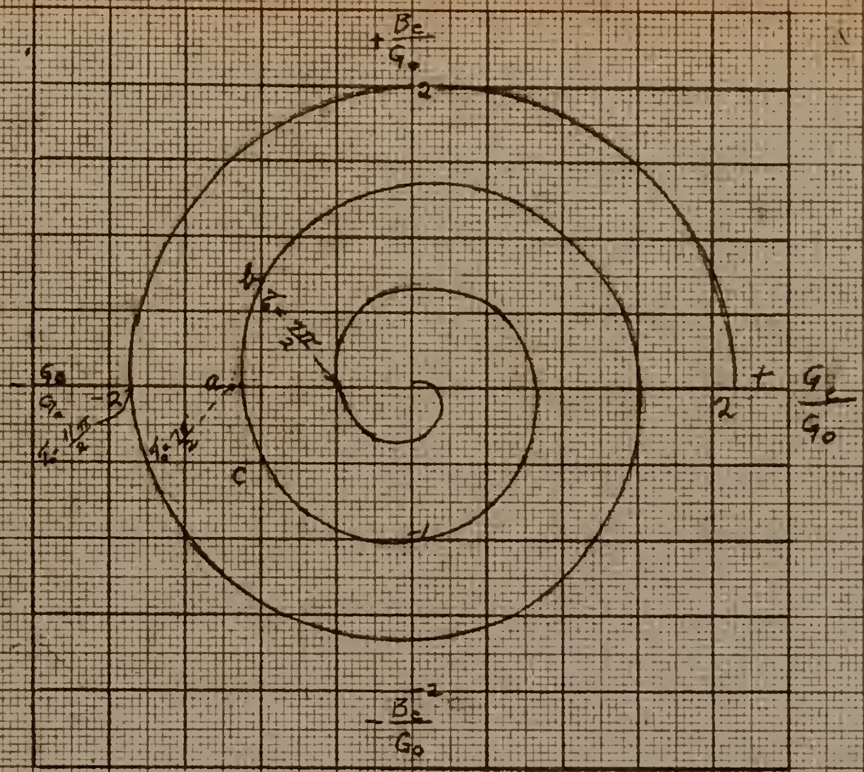

Chart of Power Output and Frequency Range of Varian Associates Klystrons

Fig 12

oscillator use that allows the tube to be frequency modulated so readily. The cavity of a reflex klystron draws a considerable current and would, therefore, require a relatively large modulating power if the modulating voltage were applied in this circuit. The repeller, on the other hand, draws no current in normal use (in fact destruction of the tube may occur if the repeller potential is allowed to become too low), and therefore requires negligible power from the modulating source. The reflex klystron, consequently, is usually modulated by impressing the modulating signal on the repeller, and it is this method that will be considered here.

In this analysis, the "small signal" or first order theory of reflex klystron operation will be used to derive modulation expressions, but will not be developed here. Reference to Ch 17 of "Vacuum Tubes" by K. R. Spangenberg[13] may be made for this first order theory.

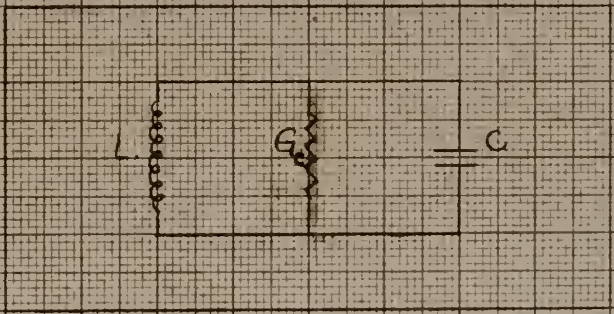
Before developing the equations for the modulation of reflex klystrons, let us consider the mechanics involved in such modulation. Suppose that the tube is operating on the center of a mode ( $\tau_0$ equal to $2\pi[n+\frac{3}{4}]$ ). Here the electron bunch returns through the cavity grids so as to average about the peak retarding field between the grids so that the fundamental component of current in the bunch is 180 degrees out of phase with the voltage between the grids. Thus the beam admittance is a negative conductance only, as shown at point "a" of the electron admittance spiral shown in Fig. 13. Now suppose that the repeller potential is made less negative so that the electrons spend a longer time in the drift space, and $\tau_0$ is increased. Now the bunch will return through the grid space after the peak retarding field so that the fundamental current in the




$$\frac{Y_e}{G_o} = A^2 \theta_o \frac{J_1(K)}{K} \epsilon^{-j(\theta_o - \frac{\pi}{2})}$$

Electronic Admittance Spiral

Fig 13




Equivalent Circuit for Klystron Cavity

Fig 14

bunch will lag behind the voltage between the grids by more than 180 degrees. The electronic admittance will then be

$$-Y_r < \frac{I\epsilon^{j\gamma}}{V_1} \quad or \quad Y_e < \frac{I}{V_1}(-\cos\gamma + j\sin\gamma)$$

so that the conductance term will (for small $\gamma$) still be negative, but a positive susceptance term is also present as shown at point "b" in Fig. 13. Because the conditions of oscillation in a reflex klystron require that the electronic admittance be equal to the circuit admittance, the power will be reduced and the frequency will decrease to re-establish the necessary equilibrium. Making the repeller potential more negative will, by the same approach, cause the beam current to lag the voltage between the grids by less than 180 degrees. The admittance will then contain a negative imaginary term as shown at point "c" in Fig. 13. This then causes the power to be reduced as before but increases the frequency generated by the tube. The use of the electronic admittance spiral in this case is not strictly correct because of the change in the pitch of the spiral under different conditions due to the non-linearity of the tube characteristics. It does, however, serve to show the trend which caused the power and frequency to change as the repeller potential is changed by a small amount.

Now the conditions for oscillations in a reflex klystron[7] are

$$Y_e + Y_c = 0 \quad ,$$

where

$$Y_e = G_0 A^2 \tau_0 \frac{J_1(A)}{A}(\sin\tau_0 + j\cos\tau_0$$

is the first order beam admittance[13] referred to the cavity grids and

$$Y_c \cong G_c(1 + j2Q\delta)$$

is the loaded cavity admittance*, not including beam loading, also

referred to the cavity grids. Following the general procedure of Jepsen[6,7] by equating the real and imaginary parts of $Y_e$ and $Y_c$ gives

$$G_0 A^2 \tau_0 \frac{J_1(A)}{A} \sin \tau_0 = G_c$$

and

$$G_0 A^2 \tau_0 \frac{J_1(A)}{A} \cos \tau_0 = 2Q\delta G_c$$

Upon eliminating $G_0 A^2 \tau_0 \frac{J_1(A)}{A}$ we obtain

$$\frac{1}{\sin \tau_0} = \frac{2Q\delta}{\cos \tau_0}, \qquad \delta = \frac{f-f_0}{f_0} = \frac{1}{2Q}\cot \tau_0$$

$$f = f_0\left(1+\frac{1}{2Q}\cot \tau_0\right) = f_0\left(1+\frac{1}{2Q}\cot \frac{4\pi f d\sqrt{2\frac{m}{e}V_0}}{V_R+V_0}\right)$$

This expression, being transcendental in f, is not **easy** to use. The modulation sensitivity, $S=\frac{df}{dV_R}$, is easier to use, and is employed extensively in klystron analysis.

$$S = \frac{f_0}{2Q}\,\frac{4\pi d\sqrt{2\frac{m}{e}V_0}\left[(V_R+V_0)\frac{df}{dV_R}-f\right]}{(V_R+V_0)^2}\left(-\csc^2 \frac{4\pi f d\sqrt{2\frac{m}{e}V_0}}{V_R+V_0}\right)$$

or

$$S = \frac{f_0}{2Q}\left(\frac{\tau_0}{V_R+V_0} - \frac{\tau_0}{f}\frac{df}{dV_R}\right)\csc^2 \tau_0$$

$$\frac{df}{dV_R}\left(1+\frac{f_0\tau_0}{2Qf}\csc^2 \tau_0\right) = \frac{f_0\tau_0}{2Q(V_R+V_0)}\csc^2 \tau_0$$

and finally

$$S = \frac{df}{dV_R} = \frac{\frac{f_0\tau_0}{2Q(V_R+V_0)}\csc^2 \tau_0}{1+\frac{f_0\tau_0}{2Qf}\csc^2 \tau_0}$$

* Represent the cavity by the approximate equivalent circuit of Fig. 14 with $\omega^2 = \frac{1}{LC}$ and $Q = \frac{\omega_0 C}{G_R}$,

$$Y_c = G_R + j\left(\omega C - \frac{1}{\omega L}\right) = G_R + j\,\omega_0 C\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right] = G_R\left[1+jQ\left(\frac{\omega-\omega_0}{\omega_0}\right)\left(\frac{\omega+\omega_0}{\omega}\right)\right]$$

but for $\omega \simeq \omega_0$, $\frac{\omega+\omega_0}{\omega} \simeq 2$

so

$$Y_c \simeq G_R\left[1+j\,2Q\delta\right]$$

where $\delta = \frac{\omega-\omega_0}{\omega}$.

The point of greatest interest is at the mode center. Operation is generally about this point and deviation from mode center is not generally large. At mode center $f = f_0$ and $\tau_0 = 2\pi(n+\frac{3}{4})$ so that $\csc^2\tau_0 = 1$. Further, Q is generally a few hundred, say 200, and n may be about 4 so that $\tau_0$ is about 30. Thus $\frac{f_0\tau_0}{2Qf}\csc^2\tau_0 \simeq \frac{30}{400} = .075$ . If this term is neglected compared to 1, the modulation sensitivity may be approximated by

$$S \simeq \frac{f_0\tau_0}{2Q(V_R+V_0)}\csc^2\tau_0 = \frac{2\pi f_0 f d\sqrt{2\frac{m}{e}V_0}}{Q(V_R+V_0)^2\sin^2\tau_0} .$$

And at mode center, at which point published modulation sensitivities are often measured,

$$S \simeq \frac{2\pi f_0^2 d\sqrt{2\frac{m}{e}V_0}}{Q(V_R+V_0)^2}$$

The second derivative of f with respect to $V_R$ is also of interest in finding the distortion produced in the process of modulating. This is

$$\frac{d^2f}{dV_R^2} \simeq \frac{d}{dV_R}\left[\frac{f_0\tau_0}{2Q(V_R+V_0)}\csc^2\tau_0\right]$$

$$= \frac{-f_0}{2Q}\left[\frac{(V_R+V_0)\left[-\frac{\tau_0\csc^2\tau_0}{V_R+V_0} + \frac{2\tau_0\csc^2\tau_0\cot\tau_0}{V_R+V_0}\right] - \tau_0\csc^2\tau_0}{(V_R+V_0)^2}\right]$$

$$= \frac{-f_0\, 4\pi f d\sqrt{2\frac{m}{e}V_0}\,\csc^2\tau_0\left[1-\tau_0\cot\tau_0\right]}{Q(V_R+V_0)^3}$$

This function, because $\lim_{\tau_0\to 0}\frac{\sin\tau_0}{\tau_0} = 1$ , goes to zero at $f = f_0$.

Higher order derivatives are required to investigate third harmonic and higher ordered distortion. However, except near $f_0$ where the second harmonic distortion becomes exceedingly small, the higher order distortion is relatively negligible.

To determine the distortion, express the reflector voltage as a D.C. voltage, $V_R'$, plus a sinusoidally varying voltage, $v = v_m \sin\omega_m t$. Then express the frequency as a Taylor expansion about v, or

$$f(V_R'+v)=f(V_R')+vf'(V_R)\Big|_{V_R'}+\frac{v^2}{2}f''(V_R)\Big|_{V_R'}+\cdots$$

Thus substituting for $v=V_m \sin\omega_m t$ and using previously obtained expressions for the first and second derivatives

$$f(V_R'+v_m\sin\omega_m t)=f(V_R')+\frac{4\pi f_o f d\sqrt{2\frac{m}{e}V_o}\ \csc^2\tau_o}{2Q(V_R'+V_o)^2}\,v_m\sin\omega_m t$$
$$-\frac{4\pi f_o f d\sqrt{2\frac{m}{e}V_o}\ \csc^2\tau_o\left[1-\tau_o\cot\tau_o\right]}{Q\,(V_R'+V_o)^3}\,v_m^2\ \sin^2\omega_m t+\cdots$$

but $\sin^2\omega_m t=\frac{1}{2}-\frac{1}{2}\cos 2\omega_m t$ so that

$$f(V_R'+v)=f(V_R')-\frac{2\pi f_o f d\sqrt{2\frac{m}{e}V_o}\ \csc^2\tau_o\left[1-\cot\tau_o\right]v_m^2}{Q\,(V_R'+V_o)^3}$$
$$+\frac{2\pi f_o f d\sqrt{2\frac{m}{e}V_o}\ \csc^2\tau_o\ v_m}{Q\,(V_R'+V_o)^2}\sin\omega_m t$$
$$+\frac{2\pi f_o f d\sqrt{2\frac{m}{e}V_o}\ \csc^2\tau_o\left[1-\tau_o\cot\tau_o\right]}{Q\,(V_R'+V_o)^3}\,v_m^2\cos 2\omega_m t+\cdots$$

and

$$D_2=\frac{v_m\left[1-\tau_o\cot\tau_o\right]}{V_R'+V_o}$$

Using the conventional means of frequency modulating a klystron by applying the modulating signal to the repeller also inherently incurs a certain amount of amplitude modulation. Following the approach of Jepsen[6,7] further, the first order theory of power output vs repeller voltage will be developed.

Now the power output of a klystron will be $P_{out}=\frac{1}{2}G_L V_1^2$ where $G_L$ is the load conductance referred to the klystron grids and $V_1$ is the peak voltage across the grids.

Using an expression from the determination of frequency as a function of $V_R$,

$$G_o A^2\tau_o\frac{J_1(A)}{A}\sin\tau_o=G_L$$

and here letting $G_c = G_L + G_R$, and solving for $V_1$ by an approximate method because of the transcendental nature of the equation.

Rearranging
$$\frac{J_1(k)}{k} = \frac{-(G_L+G_R)}{G_0 A^2 \tau_0 \sin \tau_0} \equiv g$$

Also $\frac{J_1(k)}{k}$ may be expressed by the series expansion
$$\frac{J_1(k)}{k} = \frac{1}{2} - \frac{k^2}{4} + \frac{k^4}{96} - \cdot \cdot \cdot \cdot$$

If the first two terms are used as a first order approximation
$$\frac{J_1(k)}{k} \simeq \frac{1}{2} - \frac{k^2}{4}$$

or
$$k^2 \simeq 4\left(\frac{1}{2} - g\right)$$

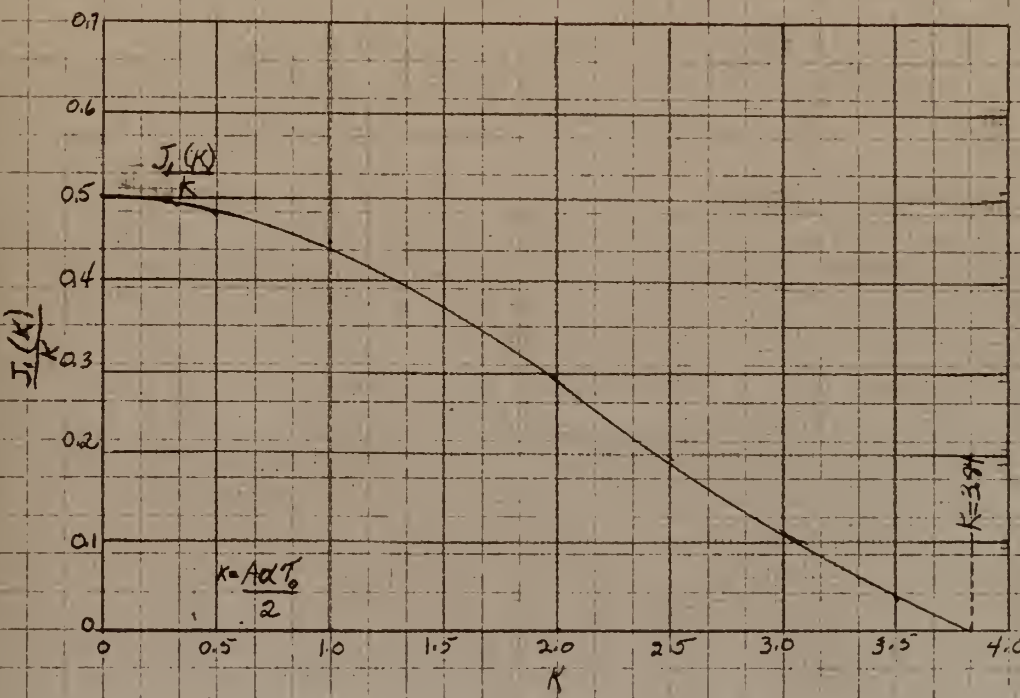
but as
$$k = \frac{A\alpha\tau_0}{2} = \frac{A V_1 \tau_0}{2 V_0}$$
$$V_1^2 = \frac{16 V_0^2}{A^2 \tau_0^2}\left(\frac{1}{2} - g\right)$$

and
$$P_{out} = \frac{8 V_0^2 G_L}{A^2 \tau_0^2}\left(\frac{1}{2} - g\right)$$

where (see Fig. 15) g lies between $\frac{1}{2}$ for k small and 0 for $k \simeq 3.84$.

When $g = \frac{1}{2}$, the power out is 0 and the klystron is just at the edge of a mode. Substituting for g,
$$P_{out} = \frac{8 V_0^2 G_L}{A^2 \tau_0^2}\left[\frac{1}{2} + \frac{G_L+G_R}{G_0 A^2 \tau_0 \sin \tau_0}\right].$$

For comparison, power output, frequency, modulation sensitivity, and second harmonic distortion are plotted as functions of the repeller voltage in Fig. 16. This plot shows that for a frequency deviation of say .0003 $f_o$ (2.25 mcs deviation at 7.425 kmcs) in the case shown, the modulation sensitivity is nearly constant at about .54 mcs/volt, the second harmonic distortion is below 0.3%, and the change in amplitude is less than 0.06%. This condition represents a reasonable deviation at which klystrons are presently operated. However, as may be seen from




Plot of $\frac{J_1(k)}{k}$ vs. k

Fig 15

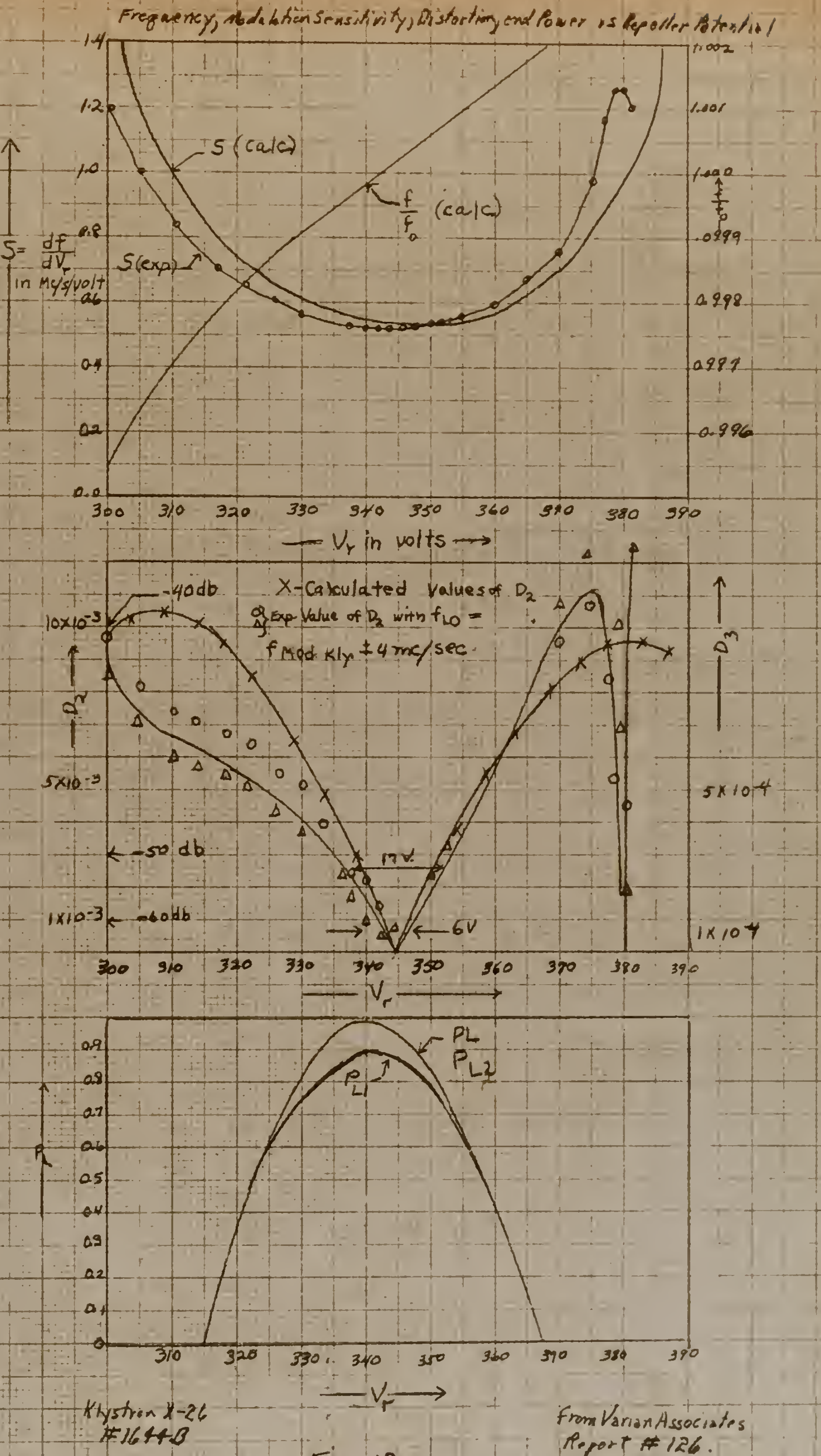



Fig 16

Modulation Sensitivity, Frequency, Second Harmonic Distortion, and Power Output vs. Reflector Potential for Reflex Klystron X-26, # 1644B

Simple Theory Parameters used for Fig. 16

Tube used was Varian Associates Reflex Klystron X-26 No. 1644B

$V_o$ = 750 volts

$I_o$ = .043 amps (current getting through r-f gap for the second time).

K = 2.1 x $10^{-6}$ amps volts$^{-3/2}$ (perveance)

$f_o$ = 7.425 x $10^9$ cycles per second

n = 3 (mode number)

$A^2$ = .35 (modulation)

$Q_L$ = 150 (hot loaded Q; determined from experimental value of modulation sensitivity at mode center.)

M = .06 mhos (characteristic admittance of resonator $=\sqrt{\frac{C}{L}}$)

$G_R$ = 1.5 x $10^{-4}$ mhos (includes cavity losses and beam loading, both primary and secondary.)

$G_L$ = 2.5 x $10^{-4}$ mhos (load conductance)

$4\pi f_o d\sqrt{2\frac{m}{e}V_o} = 25737.94$ (chosen to bring calculated and observed points of zero second harmonic FM distortion into coincidence;)

$$V_R\Big|_{D_2=0} = 344.3 \text{ volts}$$

$$\frac{V_o(G_R+G_L)}{I_o A^2} = 19.93$$

$$\frac{8V_o^2 G_L}{A^2} = 3214$$

$\Delta f$ = $10^6$ cycles/second (peak frequency deviation)

$$\frac{\Delta f\, Q_L}{f_o} = .0202.$$

Fig. 16, the deviation may be increased considerably provided that the increases in AM and distortion are tolerable.

Having considered a simplified model of the way in which a reflex klystron is frequency modulated, having followed this by an analysis showing the frequency, modulation sensitivity, and second harmonic distortion dependence upon repeller voltage, as well as a first order approximation to the way in which power output also varies when the repeller potential is varied, it is next of interest to consider the capabilities of some of the tubes presently available that may be employed in a frequency modulated microwave system. This has, in fact, been done already to a limited extent in that some experimental data is presented along with the theoretical curves in Fig. 16. Additional information here presented is based upon the data sheets published by the manufacturer.

The experimental data displayed in Fig. 16 was for a Varian Associates X-26B klystron. This klystron is one of a series of six such klystrons designated X-26A through X-26F, and covering, by means of this series, the frequency range from 5925 megacycles to 7735 megacycles except for a small band from 6425 to 6575 megacycles. The X-26A is the tube of the series with the highest frequency range. This series of tubes was designed for microwave relay operation. Typical operation is presented as follows:

| | |
|---|---|
| Frequency | 7275 mcs |
| Resonator Voltage | 750 v |
| Resonator Current | 75 ma |
| Reflector Voltage | -300 v (with respect to cathode) |
| Power Output | 1 w |
| Electronic tuning range | 35 mcs |
| Temperature Coefficient | Less than 25 kc/degree C. |
| Modulation Sensitivity | 350 kc/v |

| | |
|---|---|
| Load VSWR | Less than 1.2 |
| Harmonic Distortion with 1mc deviation for 2.5 v reflector voltage range | Below -60 db |
| for 25 v reflector voltage range | Below -40 db |

From these figures, it is seen that the input power to the resonator is about 50 watts and that the efficiency is onlyabout 2%. Because of the relatively high power lost to the cavity, plus the heating due to filament current, an air flow of about 30 cubic feet per minute is required for cooling in normal operation. The X-26 series of tubes bolt directly to a waveguide flange. Tuning is accomplished by means of a screw slotted for screwdriver adjustment.

In X band, one of the tubes now available is the Varian Associates V-290. Although this tube is primarily meant for radar local oscillator and similar use, it may quite readily be used as a microwave oscillator for communication purposes over normal "lihe-of-sight" links. This tube is a particularly rugged tube with lock nuts on the tuning screws, and silicone rubber caps on the ends where the supply leads are brought in. This construction produces a high degree of insensitivity to shock, vibration, humidity, and low pressure. This tube has a frequency range of from 8.5 kmcs to 10.5 kmcs. Typical operation from the manufacturers data sheet for this tube is given as:

| | | | |
|---|---|---|---|
| Resonator Voltage | 150 | 300 | 350 v |
| Mode | 6 3/4 | 5 3/4 | 4 3/4 |
| Frequency | 9.3 | 9.3 | 9.3 kmcs |
| Resonator Current | 15 | 42 | 52 ma |
| Reflector Voltage | -90 | -100 | -150 v |
| Power output | 12 | 48 | 140 mw |
| Electronic Tuning Range | 28 | 82 | 60 mc |
| Modulation Sensitivity | 2.5 | 1.6 | 1.2 mc/v |
| Temperature Coefficient | 60 | 60 | 60 kc/degree C. |
| Load VSWR less than | 1.1 | 1.1 | 1.1 |

For mode 4 3/4, which communication type operation would probably require, the power to the resonator is seen to be about 18 watts for an efficiency of about 0.8 percent. For operation above 10 watts input to the resonator, forced air cooling is required.

Typical data curves for the V-290 are shown in Fig. 17.

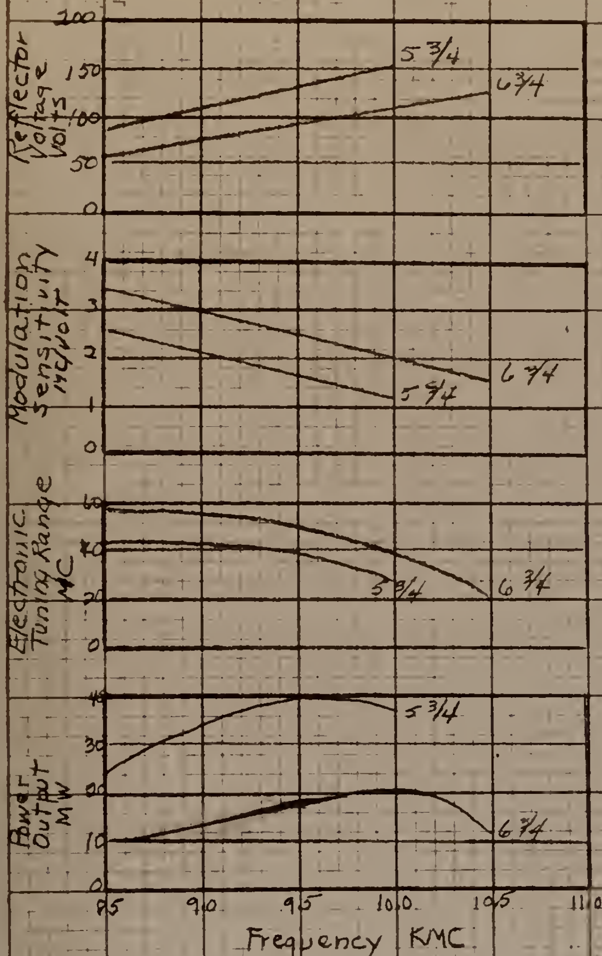
Several tubes which are electrically quite similar to the V-290 are also available for use where operating conditions are less severe. Some of these types (which carry different designating numbers) are expecially adapted for remote tuning by means of a motor so that access to the transmitter is not necessary for making adjustments.

These examples demonstrate some of the types of klystrons now available, and give an idea of the characteristics that may be expected. They by no means are meant to indicate a limitation on the capabilities of the reflex klystrons that may be obtained. A wide range of characteristics different from the examples just given may be obtained either from present standard production of manufacturing companies, or by special design and construction contracts with these companies whereby tubes may be modified or redesigned to meet special requirements. However, the price of klystrons, even for the standard production tubes, is many times that of conventional electron tubes of comparable sizes.

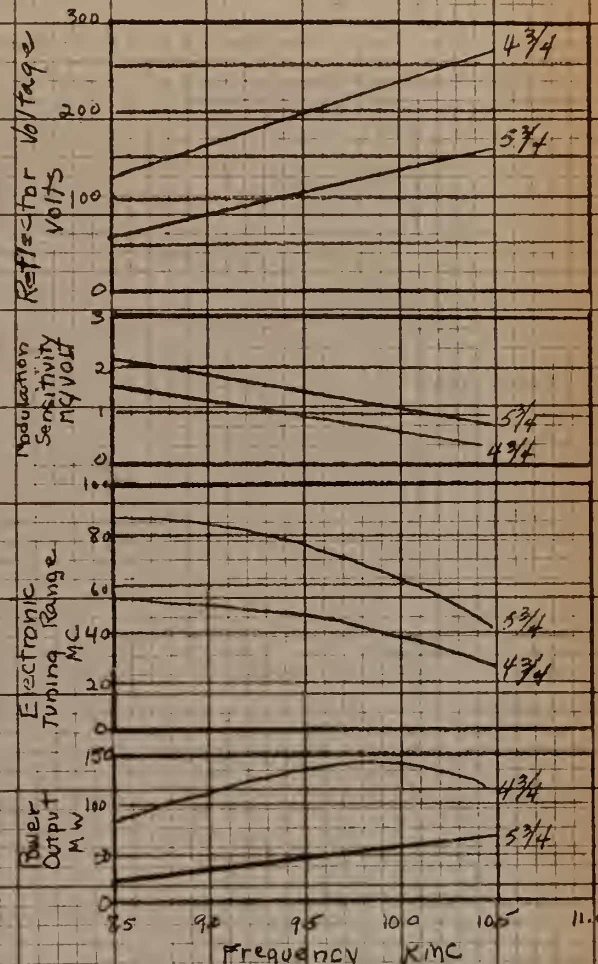
From the foregoing then, it is seen that the reflex klystron is quite capable of serving as the frequency generating source, or further as the power source, for frequency modulated communication systems in the microwave region. Reflex klystrons tend to be quite sensitive to supply voltage variations, and thus require well regulated supplies. Amplitude modulation occurs with frequency modulation unless the deviation

Typical Data Curves

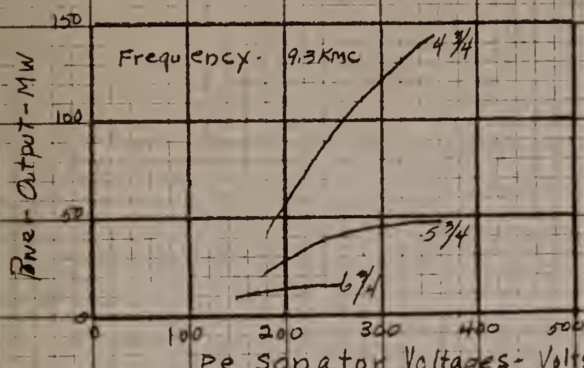
for Varian V-290 Reflex Klystron



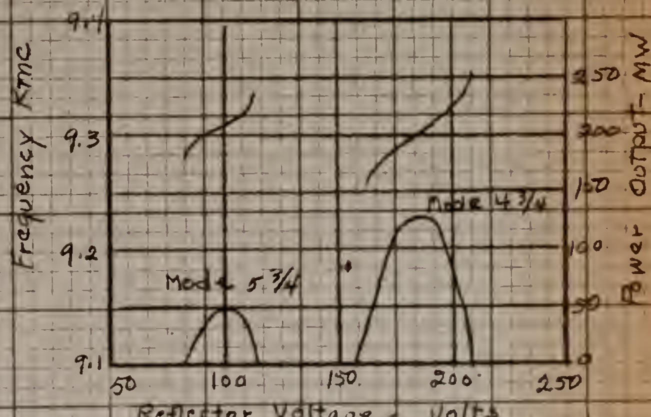

Typical Data $E_{rs}$ = 200 V




Typical Data $E_{rs}$ = 300 Volts




Power Output vs Resonator voltage




Reflector Voltage Mode Characteristics $E_{rs}$ = 350 Volts

Fig 17

is kept small. They are not efficient, and even for relatively low output powers, may require auxiliary cooling systems. However, they are easily modulated, and require a negligible modulating power. Also the tubes may be made small and rugged. Reflex klystrons are widely used today in relay and other microwave communication work, and will undoubtedly be used extensively in new communication devices in the future.

CHAPTER IV

# FREQUENCY MODULATION OF A BACKWARD WAVE OSCILLATOR

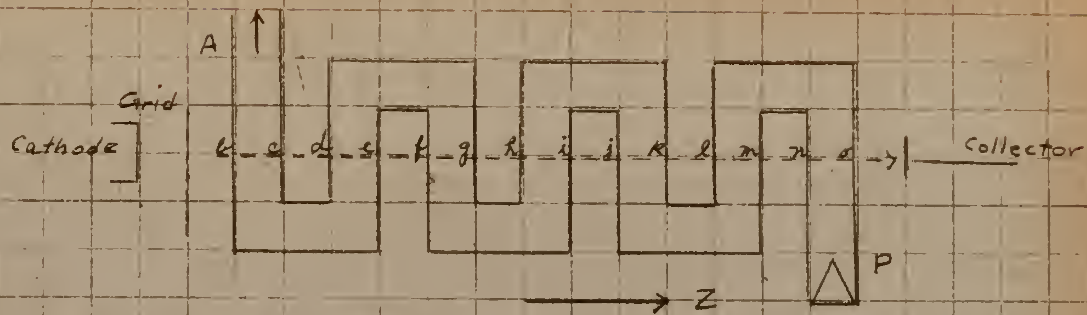
One recent microwave tube development that has aroused the interest of the industry is the traveling-wave tube. In its basic conception, this tube is an amplifier, generally quite broad-banded, and capable of relatively high gain. Its operation, similar to that of the klystron, depends upon a bunching of the electron beam from a gun system, and the interaction of the beam with an r.f. field; but unlike the klystron, bunching and interaction occur either continually or periodically over an extensive length of beam, a "slow wave" structure being used in effect, to reduce the phase velocity of the wave to approximately that of the electron beam. This traveling-wave amplifier may have the output coupled back through an external circuit to the input. The tube is then capable of oscillation provided that the combined internal and external path sum to an integral number of wave lengths, and that the beam current is equal to or greater than a given "starting current". The frequency of this oscillator may then be changed by changing the effective length of either the internal or external path.

Yet another variation of the traveling-wave tube is the backward wave oscillator[8] (or backward beam oscillator, depending upon your point of view). This tube shows considerable promise as a microwave generator which will be easy to frequency-modulate, and especially one in which the frequency deviation may be made quite large in order to trade bandwidth for an increase in signal-to-noise ratio if so desired.[3] It is this version of the traveling-wave tube, the backward wave oscillator, which will be considered here.

The backward wave oscillator may employ an electron gun assembly similar to that used in a cathode ray tube, except that the current is generally many times larger. The beam from this gun travels through an r.f. structure where interaction between the beam and the fields may take place. This structure must be periodic in nature as seen by the electron beam. After traversing the length of the structure, the remaining electrons are intercepted by a collector electrode. The entire r.f. structure operates at a positive potential relative to the cathode. The collector is operated at a potential at or near that of the r.f. structure. A separate grid between the r.f. structure and the cathode may be used to control the amount of beam current.

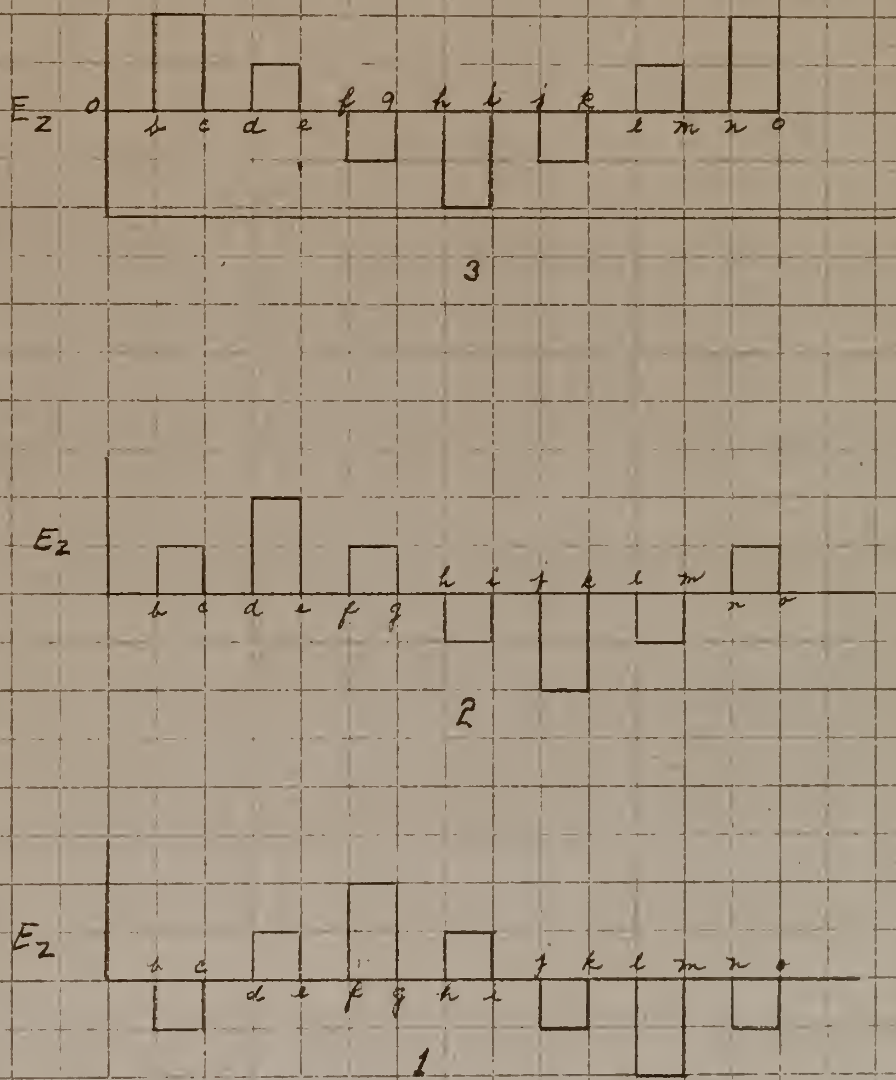
The r.f. structure may be any one of a large number of designs.[11] Most of the discussion here will deal with a "bi-filar helix" structure. However, the "folded waveguide" structure will be used in the first explanations because of its simplicity.

Consider the folded waveguide structure of Fig. 18. Here a wave at "F" in the guide would travel through the guide toward "A" with a speed near that of light. Its velocity along the axis, such as from "e" to "c" however, would be reduced by a factor equal to the periodicity of the structure divided by the guide length that the wave travelled. In this way the effective velocity of the wave may be reduced to about one-tenth the speed of light so that the wave moves at about the same speed as an electron beam, and may interact with it. Also, it will be noted, the field is not continuous down the axis of the folded structure, but rather, the field intensity may be higher between "b" and "c", and between "d" and "e", but will be virtually




Schematic Sketch of Folded Waveguide Backward Wave Oscillator

Fig 18



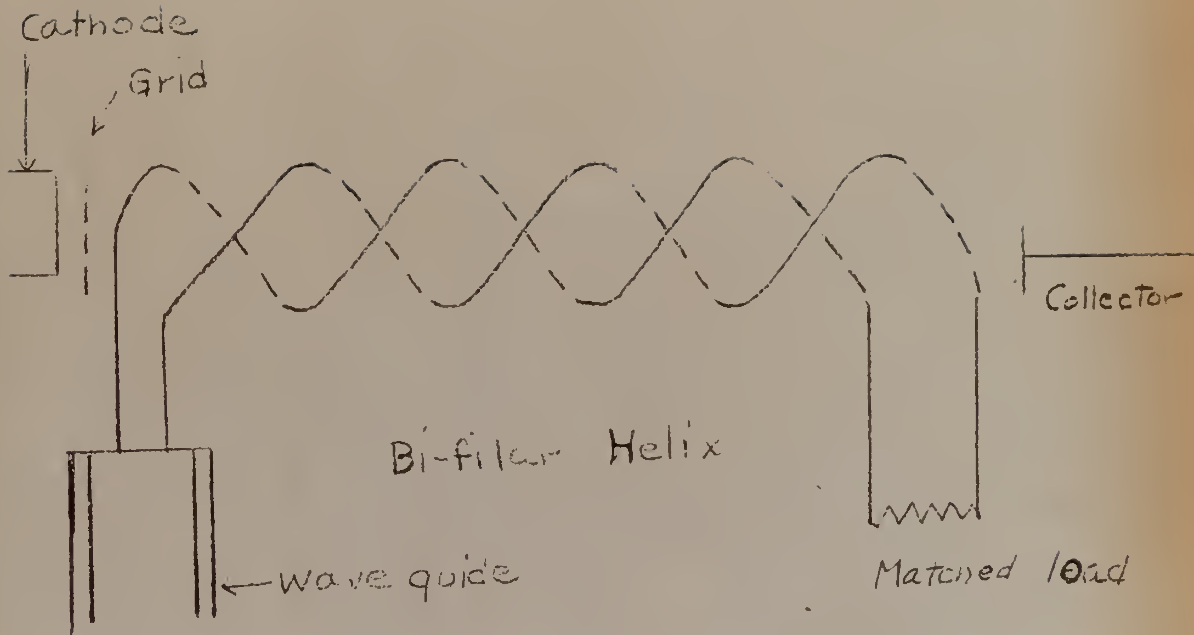

Sketches of Axial Field Intensity for Folded Waveguide Backward Wave Oscillator

Fig 19

non-existent between "c" and "d" outside the guide. Thus a plot of the field down the axis of the structure might be, at a given instant like that shown in Fig. 19-1, and parts of an r.f. cycle later as shown in Fig. 19-2 and Fig. 19-3. This field may be analyzed by a complex Fourier analysis to give both positive and negative" space harmonics" with phase constants of $\beta_n = \beta_0 + \frac{2\pi n}{p}$ where n is zero or a positive or negative integer, and p is the periodicity of the structure. The backward wave oscillator depends upon beam interaction with a negative or backward component, and thence comes its name.

With the bi-filar helix r.f. structure to be considered in this analysis, the action is quite similar to that of the folded waveguide except that the field periodicity exists only off the axis of the helix. That is, only the off axis electrons are involved in interaction with the field, and the farther off the axis and the nearer to the helix, the greater is this interaction. A sketch of the bi-filar structure is shown in Fig. 20. In effect, this structure is a parallel wire transmission line wound in the form of a solenoid. The units constructed to date at Varian Associates have been enclosed in glass tubing for support. A short section of parallel line connects the helix to the waveguide at the cathode, or output, end of the helix, and a resistive load is used at the collector end to prevent reflection of forward waves.

Equations may be set up[4] for the effect of the field upon the electron beam neglecting the interaction of the beam back on the circuit, and for the field impressed upon the circuit due to the r.f. components of current in the beam, neglecting again the back interaction. These equations may then be solved simultaneously to give the operating conditions. It is not the purpose of this paper to consider these conditions, therefore




Backward wave oscillator
Schematic diagram

Fig 20

only the results from this process which are necessary for frequency computations will be given here. The results obtained by Heffner[4] which are of use to us at this time may be stated in equation form as:

$$(\beta - \beta_e)L = 3.00$$

where L is the helix length, for the first negative spatial harmonic (n equal to -1 in the Fourier analysis). Although oscillation is also possible with higher negative space harmonics, greater current is required, and power output is less. Only the first backward component will be considered here.

Beam interaction with the wave may be considered physically as follows: Consider the actual primary wave as traveling down the helix toward the cathode. The working electrons will cross between wires of the helix against an average retarding field. Those electrons which experience a greater retarding field will be slowed down more, while those experiencing a lesser retarding field will be slowed down little. Also those electrons which experience an accelerating field will be speeded up. The net result is that the electrons tend to bunch. These bunches, then, in normal operation will, on the average, be between the wires in a region of strong field during the time that a retarding field exists between the wires, and will be beside the wires in a region of weak field, when an accelerating field exists between the wires. Thus the bunches move in syncronism with the wave, giving up energy during one retarding half cycle, being shielded by the wires during the following accelerating half cycle, and then moving between the next two wires during the next retarding half cycle. The result is then much the same as with a stroboscope which, under proper timing

conditions, makes the rotating disk appear to go slowly in a direction opposite to its actual rotation. It is this syncronization action which, when analyzed by the Fourier series method, produces the effect called spatial harmonics, or in this case, the first negative space harmonic.

Again turning to the equation $(\beta-\beta_e)L=3.00$ for the purpose of determining the frequency as a function of the beam, or helix-cathode potential, $V_o$, it is advantageous to note that $\beta L$ and $\beta_e L$ are both quite large in practice compared to 3 so that $\beta \cong \beta e$. But as discussed before

$$\beta = \beta_o + \frac{2\pi n}{p}$$

or

$$\beta = \beta_o - \frac{2\pi}{p} \quad \text{for } n = -1$$

Now $\frac{2\pi}{p} > \beta_o$ so that $\beta$ is negative and

$$\beta e = \frac{2\pi}{p} - \beta o$$

where

$$\beta o = \frac{\omega}{v_z} = \frac{2\pi a \omega}{p v_h} = \frac{2\pi a}{p} k'$$

so that

$$\beta e = \frac{2\pi}{p}(1 - k'a)$$

also $\beta e = \frac{\omega}{u_o}$ where $u_o = \sqrt{2 \frac{e}{m} V_o}$

thus giving

$$\frac{f}{\sqrt{2 \frac{e}{m} V_o}} = \frac{1}{p}(1 - k'a)$$

a form which is often valuable for analysis itself. However here we want an explicit expression for frequency, so write

$$\frac{f}{\sqrt{2 \frac{e}{m} V_o}} = \frac{1}{p} - \frac{2\pi f a}{v_h p}$$

and finally

$$f = \frac{1}{\frac{p}{\sqrt{2 \frac{e}{m} V_o}} + \frac{2\pi a d'}{c}}$$

where d' is the dielectric loading factor and c is the speed of light.

The dielectric loading factor, d', is the necessary modifying factor for the velocity of the wave due to the presence of a dielectric material near the helix. In the case of the tape helix model generally employed to obtain an approximate solution of the fields for round wire helises, and provided that the helix is enclosed in a glass tube (as is the case in the tubes being considered here) the helix is taken as an infinitely thin, perfectly conducting, tape with glass on one side and vacuum on the other side. Because of the presence of the glass, the wave does not travel as fast as it would if the helix were entirely in free space. This factor has been dealt with in a general way by Tien.[14] Another method for obtaining this factor, less elegant but also considerably less involved, and which has given reasonably good results in practice, is that method used by Dr. William Beaver of Varian Associates. This method considers the media on the vacuum side of the helix and the dielectric side of the helix to act as two capacitances in parallel. The velocity, $v_h$, is proportional to $1/\sqrt{LC}$ where $C$ is approximately $C'\left(\frac{1+\epsilon'}{2}\right)$, and where C' is the capacitance of the vacuum side only and $\epsilon'$ is the relative permittivity of the dielectric. From this, then, it is seen that the velocity is reduced by the factor $\sqrt{\frac{2}{1+\epsilon'}}$, which is herein designated d'. For cold glass, $\epsilon' = 5.1$. Then for a helix enclosed in a glass tube

$$d' \cong \frac{1}{1.75}$$

Returning again to the frequency equation, let us obtain the equation for modulation sensitivity, $S = \frac{df}{dV_0}$. This may be done by noting that

$$\frac{df}{dV_0} = \frac{df}{dV_0^{-1/2}} \frac{dV_0^{-1/2}}{dV_0} \quad \text{where} \quad \frac{dV_0^{-1/2}}{dV_0} = -\frac{1}{2} V_0^{-3/2}$$

Then

$$\frac{df}{dV_0^{-1/2}} = \frac{-\frac{p}{\sqrt{2\frac{e}{m}}}}{\left[\frac{p}{\sqrt{2\frac{e}{m}}V_0} + \frac{2\pi a d'}{c}\right]^2} = \frac{pf^2}{\sqrt{2\frac{e}{m}}}$$

and finally

$$S = \frac{df}{dV_0} = \frac{1}{2}\frac{pf^2}{\sqrt{2\frac{e}{m}V_0^3}}$$

Further, to obtain the second derivative of f with respect to $V_0$,

$$\frac{d^2f}{dV_0^2} = \frac{dS}{dV_0} = \frac{p}{2\sqrt{2\frac{e}{m}}}\frac{d\left(f^2V_0^{-3/2}\right)}{dV_0} = \frac{p}{2\sqrt{2\frac{e}{m}}}\left[2fV_0^{-3/2}\frac{df}{dV_0} - \frac{3}{2}f^2V_0^{-5/2}\right]$$

$$= \frac{p}{2\sqrt{2\frac{e}{m}}}\left[\frac{pf^3}{V_0^3\sqrt{2\frac{e}{m}}} - \frac{3}{2}\frac{f^2}{V_0^{5/2}}\right]$$

or

$$\frac{d^2f}{dV_0^2} = S\left[\frac{pf}{\sqrt{2\frac{e}{m}V_0^3}} - \frac{3}{2V_0}\right]$$

If we now let $V_0$ be made up of a D.C. voltage, $V_0'$, and a superimposed A.C. voltage $v = v_m \sin\omega_m t$, and then, as was done in the klystron analysis, expand f as a Taylor series function of $V_0$ about v we obtain

$$f(V_0'+v) = f(V_0') + v f'(V_0)\Big|_{V_0'} + \frac{v^2}{2}f''(V_0)\Big|_{V_0'} + \cdots$$

which upon substitution of equivalent quantities becomes

$$f(V_0'+v_m\sin\omega_m t) = f(V_0') + S\Big|_{V_0'}v_m\sin\omega_m t + S\left[\frac{pf}{\sqrt{2\frac{e}{m}V_0^3}} - \frac{3}{2V_0}\right]\Bigg|_{V_0'}v_m^2\sin^2\omega_m t + \cdots$$

but $\sin^2\omega_m t = \frac{1}{2} - \frac{1}{2}\cos 2\omega_m t$ so that

$$f(V_0'+v) = f(V_0') - \frac{S}{2}\left[\frac{pf}{\sqrt{2\frac{e}{m}V_0^3}} - \frac{3}{2V_0}\right]\Bigg|_{V_0'}v_m^2 + S\Big|_{V_0'}v_m\sin\omega_m t - \frac{S}{2}\left[\frac{pf}{\sqrt{2\frac{e}{m}V_0^3}} - \frac{3}{2V_0}\right]\Bigg|_{V_0'}v_m^2\cos 2\omega_m t + \cdots$$

Then the second harmonic distortion is

$$D_2 = \frac{v_m}{2}\left[\frac{3}{2V_0'} - \frac{pf}{\sqrt{2\frac{e}{m}(V_0')^3}}\right] = v_m\left[\frac{3}{4V_0'} - \frac{S}{f}\right]$$

Thus, for example if $V_0' = 1000$ volts for the VA-130 tube, the center frequency would be about 10,270 mcs., and if $v_m$ were about 50 volts, the deviation would be about 90 mcs. and the second harmonic distortion would be about 2.5 . This appears to be quite good for such a large deviation compared to presently used generators of other types.

No further theoretical figures will be given at this time. However, theoretical curves based upon the equation developed above will be considered along with the experimental data available on the tubesto be discussed.

As was stated before, backward wave oscillators are relatively new devices. For the most part, these tubes are in the developmental stage, and none are known to be commercially available at this time. The two tubes to be discussed in particular in this paper are research tubes built by the Research and Development Section of Varian Associates for the purpose of furthering their knowledge concerning such tubes. Only one copy of each type was produced, and it is not expected that they will be duplicated in their present form. Also, because these tubes are research tubes, no particular attempt was made to design characteristics into them that would be desirable in frequency modulated microwave oscillators for communication purposes. Further, and for the same reason given above, the circuitry necessary to operate these tubes is not considered satisfactory for such operation. An example of this incompatibility is the large, high-power D.C. coil into which the tube is placed, and which is necessary to keep the electron beam focused during its travel down the helix. These undesirable features can be expected to be reduced in number and magnitude of effect, or to be eliminated entirely as the tube is developed, and as the system in which the tube might operate is engineered.

Even though these tubes are relatively new, and in spite of their not being designed as frequency modulated sources, a consideration of their operation and a comparison with theoretical data will serve

nicely as a basis upon which to judge the possibilities of the backward wave oscillator in communication type systems. There are two such tubes to be considered. They are the VA-180, designed for a center frequency of 10 kmcs., and the VA-181, designed for a center frequency of 15 kmcs.

The VA-180, serial number 1, had an r.f. structure in the form of a bi-filar helix of .007 inch platinum clad wire enclosed in a glass tube. The helix internal diameter was .081 inches, which gave it a pitch diameter of .088 inches. The structure was 3.9 inches in total length, and each wire made 23 turns per inch. As stated before, the center frequency was designed to be 10 kmcs. Although this tube was evidently damaged in manufacture (inspection after disassembly disclosed that the glass had been broken near the cathode end, and that the rest of the helix was slightly offset), the tube operated reasonably well. As far as its frequency vs beam potential characteristics were concerned, they appeared to be extremely good in that output was obtained continuously as a function of the beam potential from about 7 kmcs to about 12 kmcs. Also, the measured characteristics followed very close to the theoretical calculations as shown for a portion of the tube's range in Fig. 21. Fig. 22 shows some of the characteristics of the tube when operated under pulsed conditions (which was done in order to keep the current, and thus the heating, down in this experimental tube). It will be noted that the frequency vs beam potential curve is shifted somewhat from that shown in Fig. 21. One major factor for this is the additional beam loading due to the beam current being greater in the case of Fig. 22. Theoretical curves showing the modulation sensitivity and second harmonic distortion as functions of beam voltage are shown in Fig. 23.

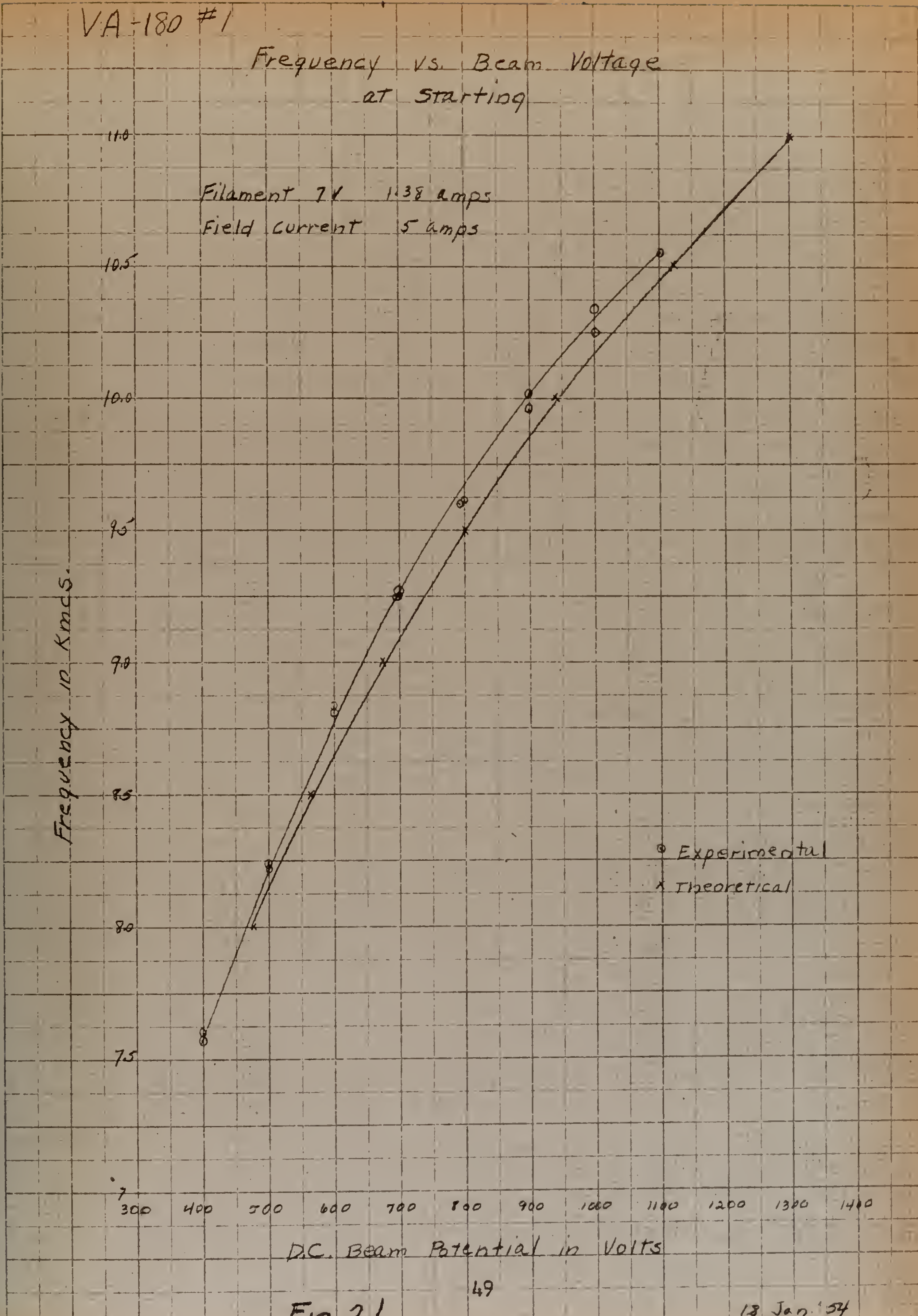
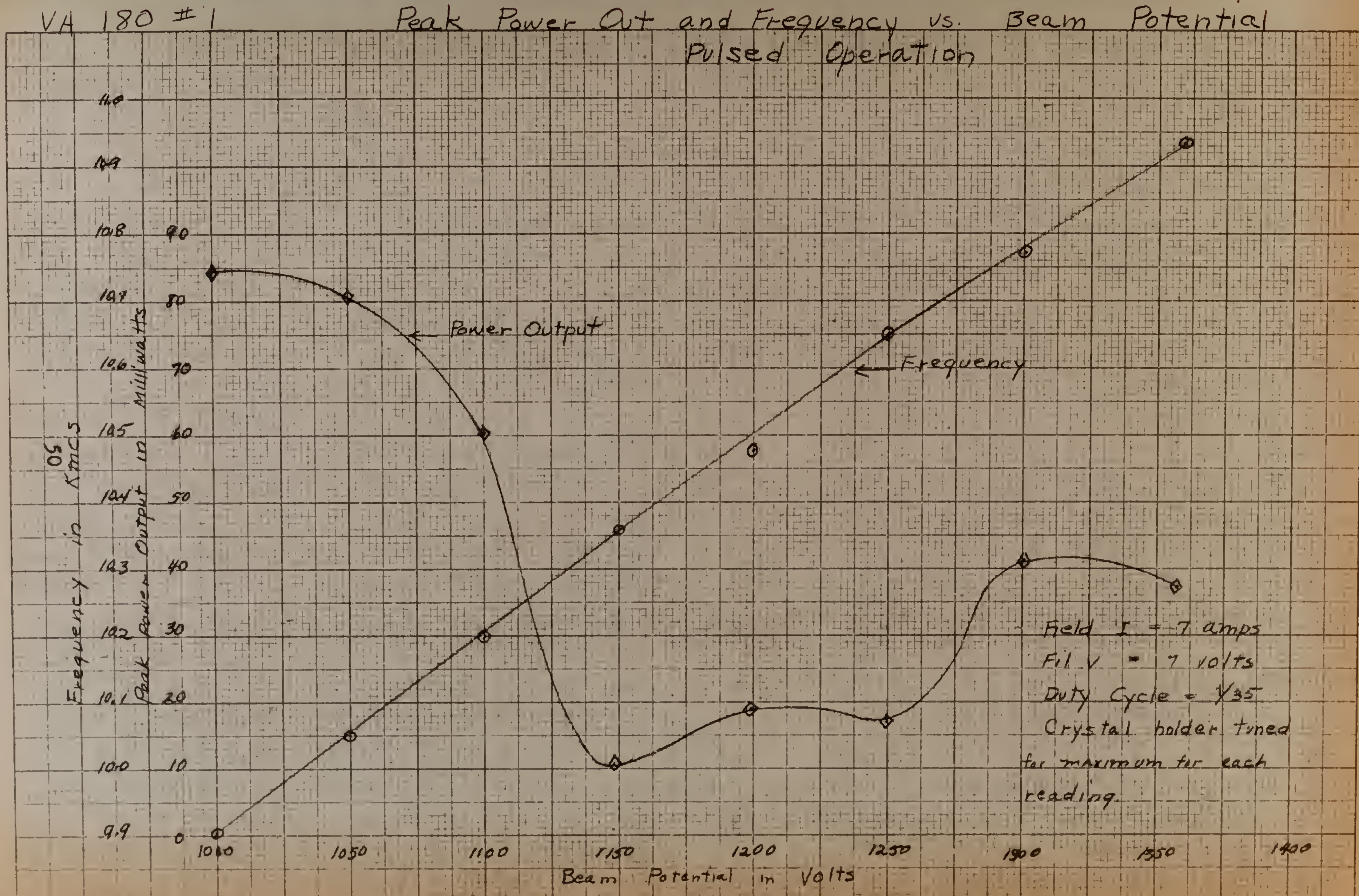

VA-180 #1
Frequency vs. Beam Voltage
at Starting
Filament 7V 1.38 amps
Field Current 5 amps
Frequency in Kmcs.
11.0
10.5
10.0
9.5
9.0
8.5
8.0
7.5
7
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
D.C. Beam Potential in Volts
Experimental
Theoretical
18 Jan. 54

Fig 21



VA 180 #1
Peak Power Out and Frequency vs. Beam Potential
Pulsed Operation
Power Output
Frequency
Frequency in Kmcs
Peak Power Output in Milliwatts
11.0
10.9
10.8
10.7
10.6
10.5
10.4
10.3
10.2
10.1
10.0
9.9
90
80
70
60
50
40
30
20
10
0
1000
1050
1100
1150
1200
1250
1300
1350
1400
Beam Potential in Volts
Field I = 7 amps
Fil V = 7 volts
Duty Cycle = 1/35
Crystal holder tuned
for maximum for each
reading.


Fig 22

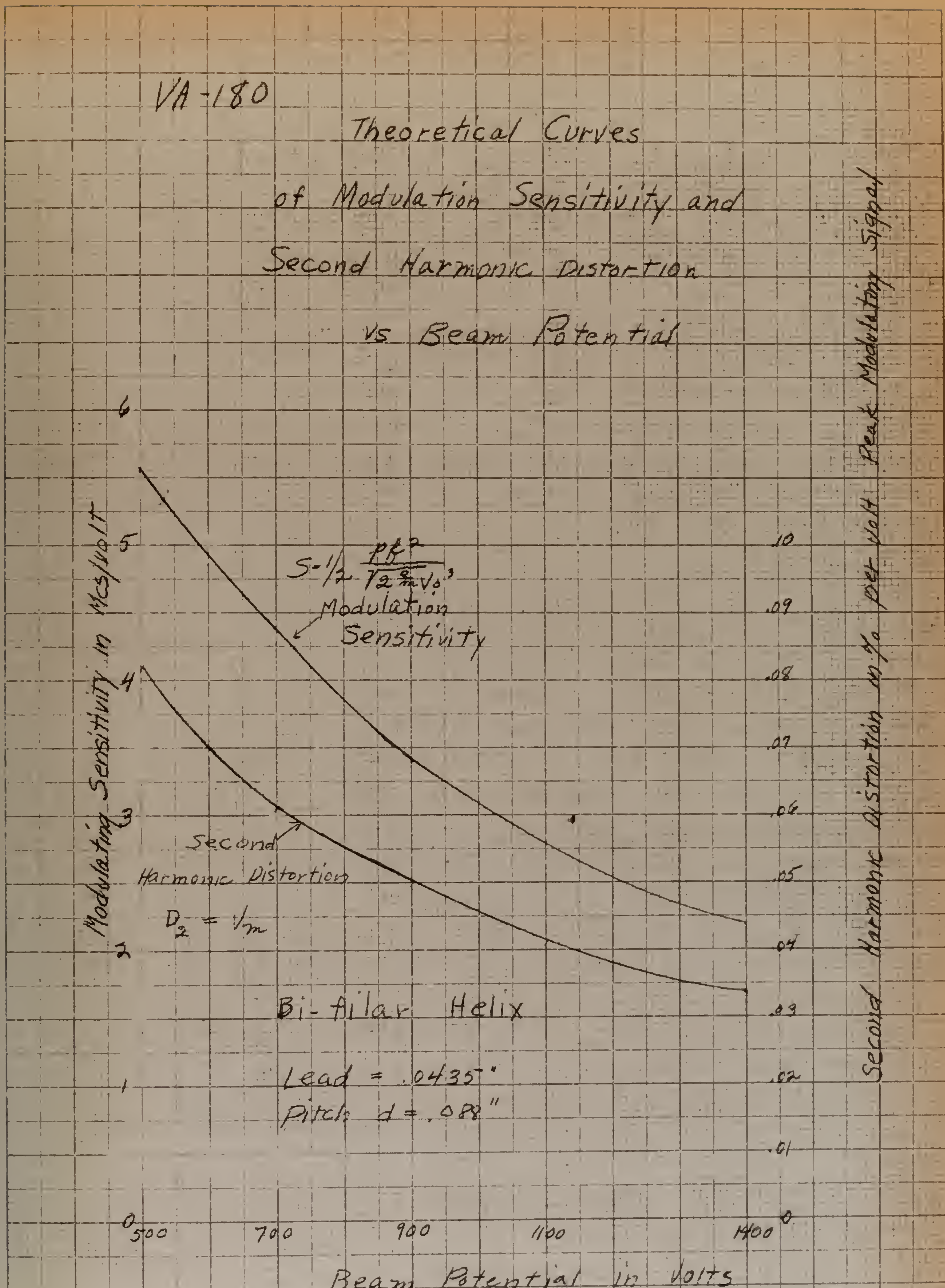

VA-180
Theoretical Curves
of Modulation Sensitivity and
Second Harmonic Distortion
vs Beam Potential
$S = \frac{1}{2}\frac{p\ell^2}{\sqrt{2\frac{e}{m}V_0^3}}$
Modulation
Sensitivity
Second
Harmonic Distortion
$D_2 = 1/m$
Bi-filar Helix
Lead = .0435"
Pitch d = .088"
Modulating Sensitivity in Mcs/volt
Second Harmonic Distortion in % per volt Peak Modulating Signal
Beam Potential in Volts
0
1
2
3
4
5
6
500
700
900
1100
1400
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10

Fig 23

The VA-181, serial number 1, is quite similar to the VA-180 as far as the r.f. structure is concerned. Again the bi-filar helix of .007 inch platinum clad wire enclosed in a glass tube is used. The helix internal diameter is .052 inches, which gives a pitch diameter of .059 inches. The lead is .0278 inches, or in other words, each wire makes 36 turns per inch. The design center frequency is 15 kmcs.

Again, as far as the frequency dependence upon the beam potential is concerned, the tube operated quite satisfactorily. Fig. 24 demonstrates both the relatively good linearity (over a range of a few hundred megacycles or less) and the small departure from the theoretical curve. Fig. 25 shows an extension of the theoretical curve over a range greater than the design frequency range so that the nature of the tuning curve may be seen more easily. Fig. 26 shows the theoretical curves for modulation sensitivity and second harmonic distortion as a function of the beam voltage for the VA-181.

From the foregoing, it may be seen that backward wave oscillators, both from a theoretical consideration and from the experimental data, have very desirable frequency modulation characteristics with the possibility of giving considerably higher deviation with reasonably low distortion than can be obtained from other sources at the present time. Then too, although some modulating power is required, generally this power is small compared to the D.C. power of the beam. Further, although the power output varies considerably over the entire range that a backward wave oscillator may operate, (see Fig. 27 for VA-181) the amount of variation will probably be reduced considerably as better coupling circuits are designed; and even with the present tubes,

VA-181 #1

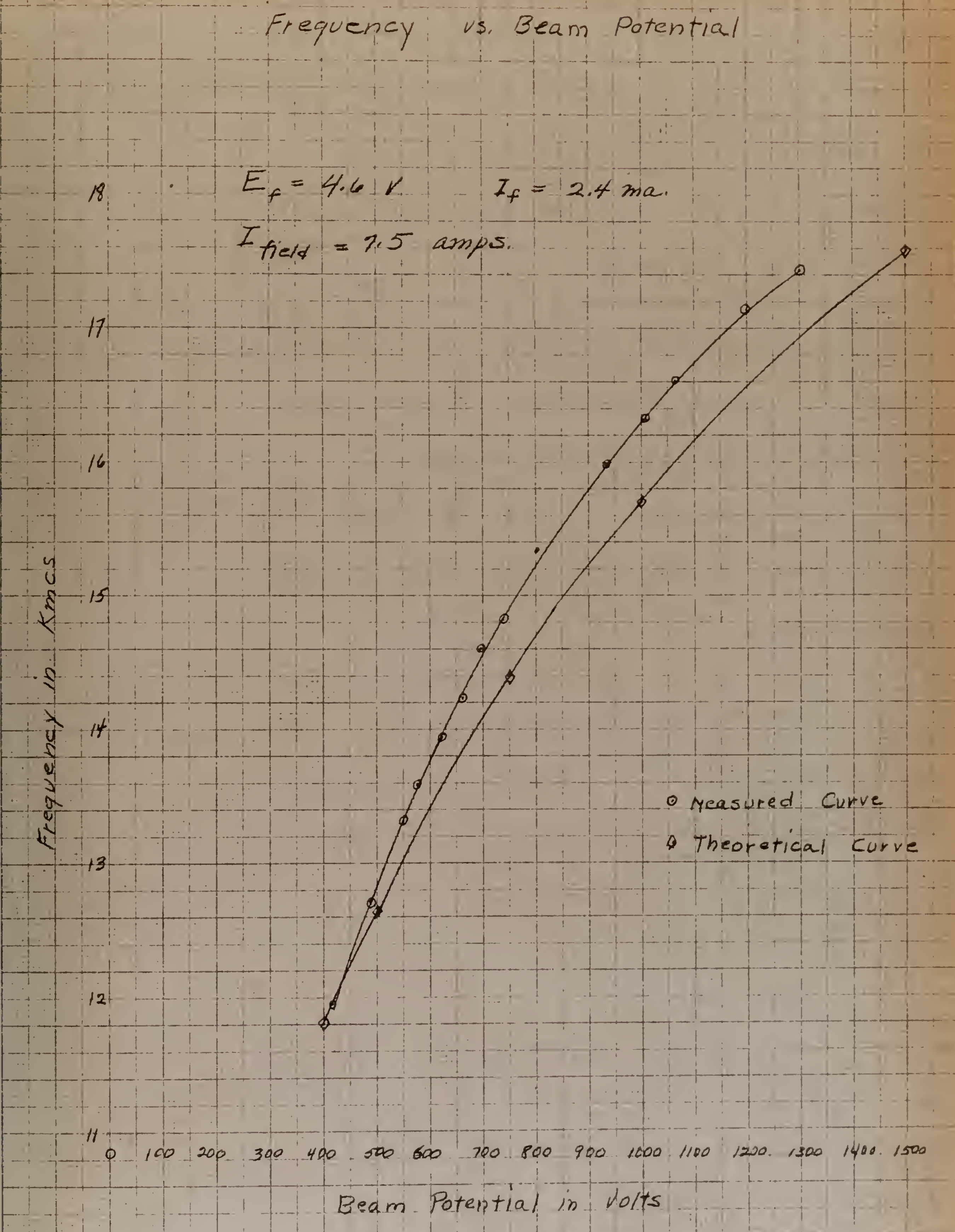



Fig 24



12 Mar. '54

VA-181

Frequency vs. Beam Voltage

Theoretical

Extended Curve

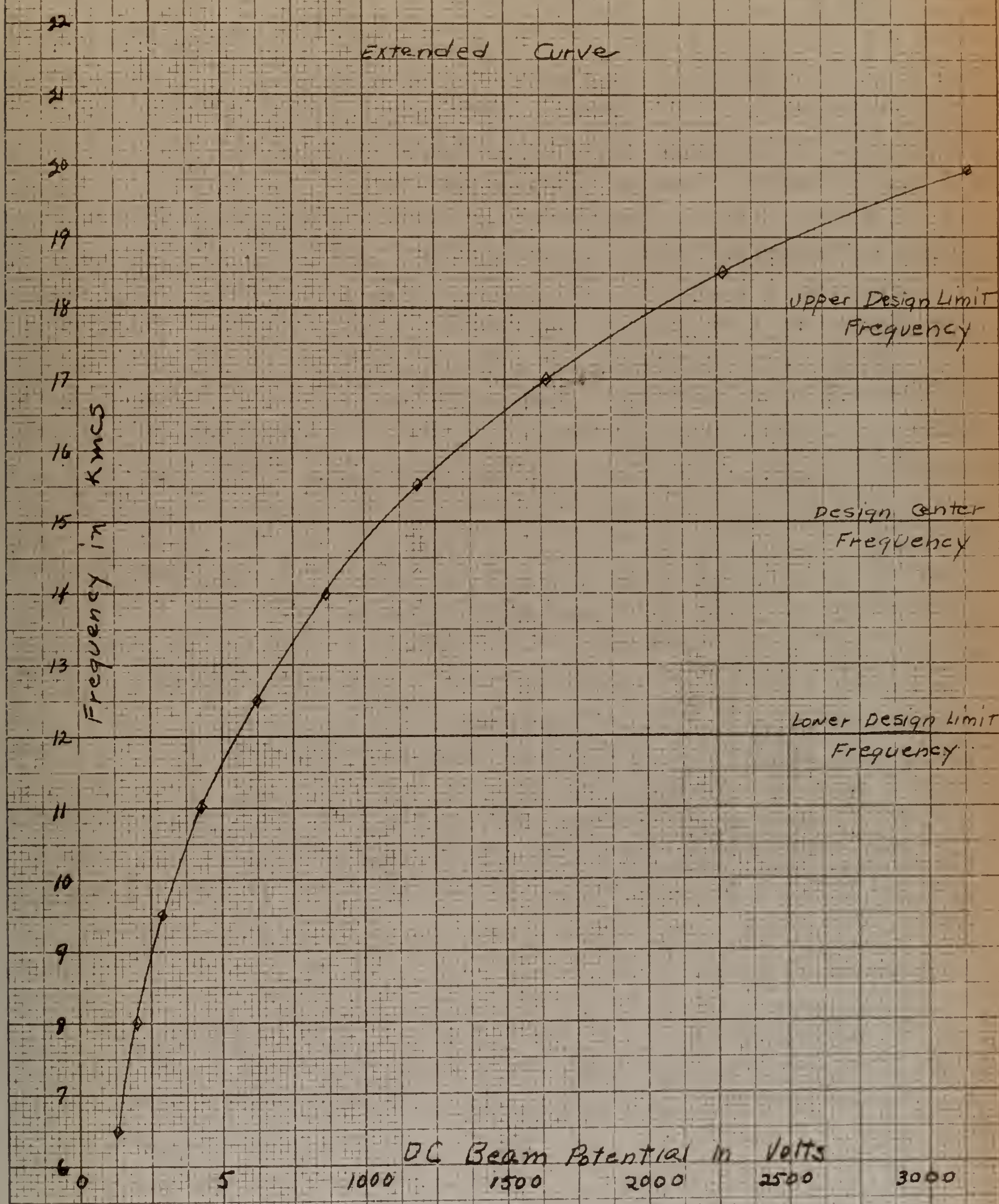
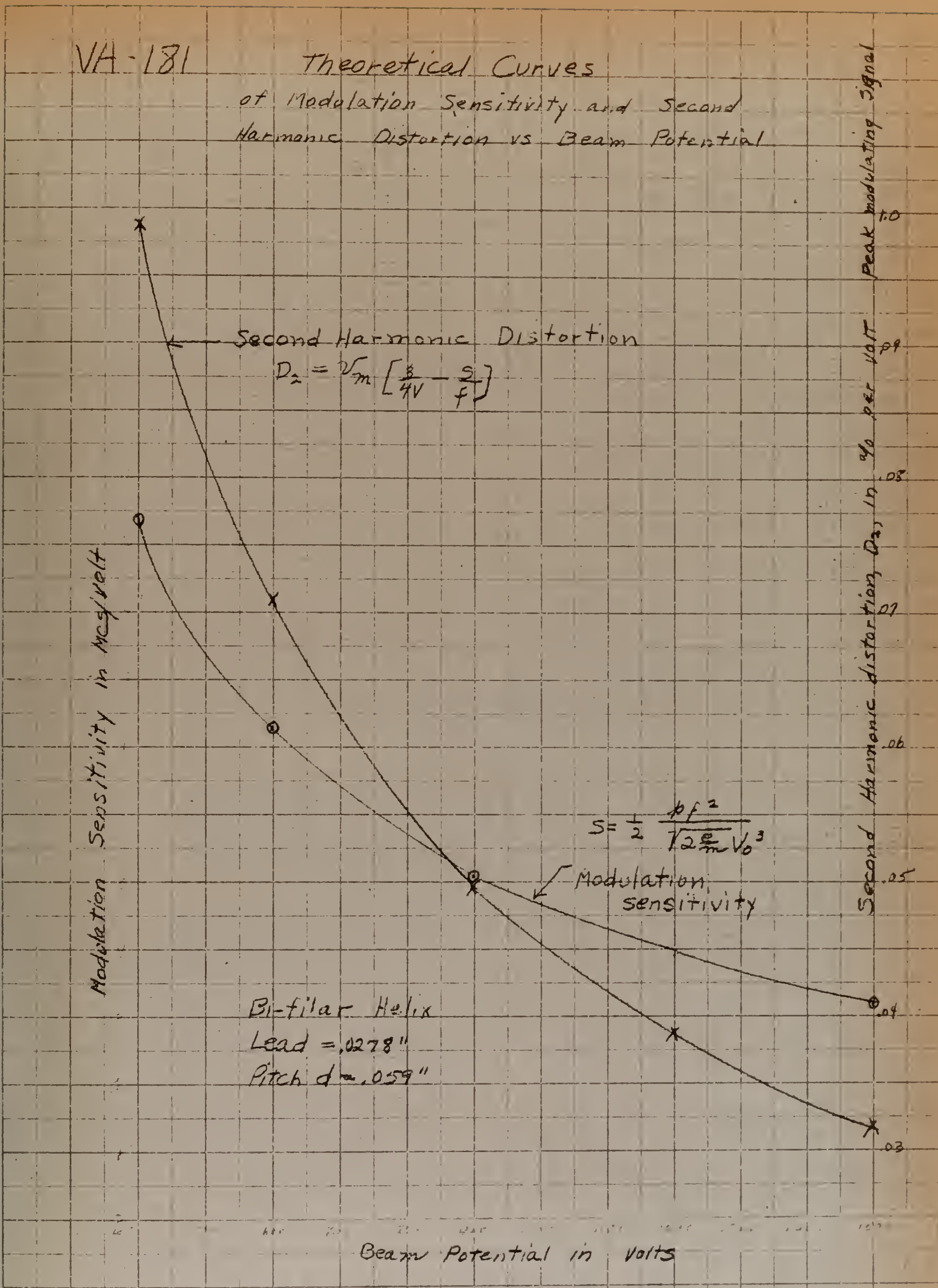



Fig 25

54

16 Feb '5



VA-181
Theoretical Curves of Modulation Sensitivity and Second Harmonic Distortion vs Beam Potential
Second Harmonic Distortion
$D_2 = V_m \left[ \frac{s}{4V} - \frac{s}{f} \right]$
$S = \frac{1}{2} \frac{pf^2}{\sqrt{2\frac{e}{m}V_0^3}}$
Modulation sensitivity
Bi-filar Helix
Lead = .0278"
Pitch d = .059"
Modulation Sensitivity in Mcs/volt
Second Harmonic distortion $D_2$, in % per volt Peak modulating Signal
1.0
.09
.08
.07
.06
.05
.04
.03
Beam Potential in Volts


Fig 26

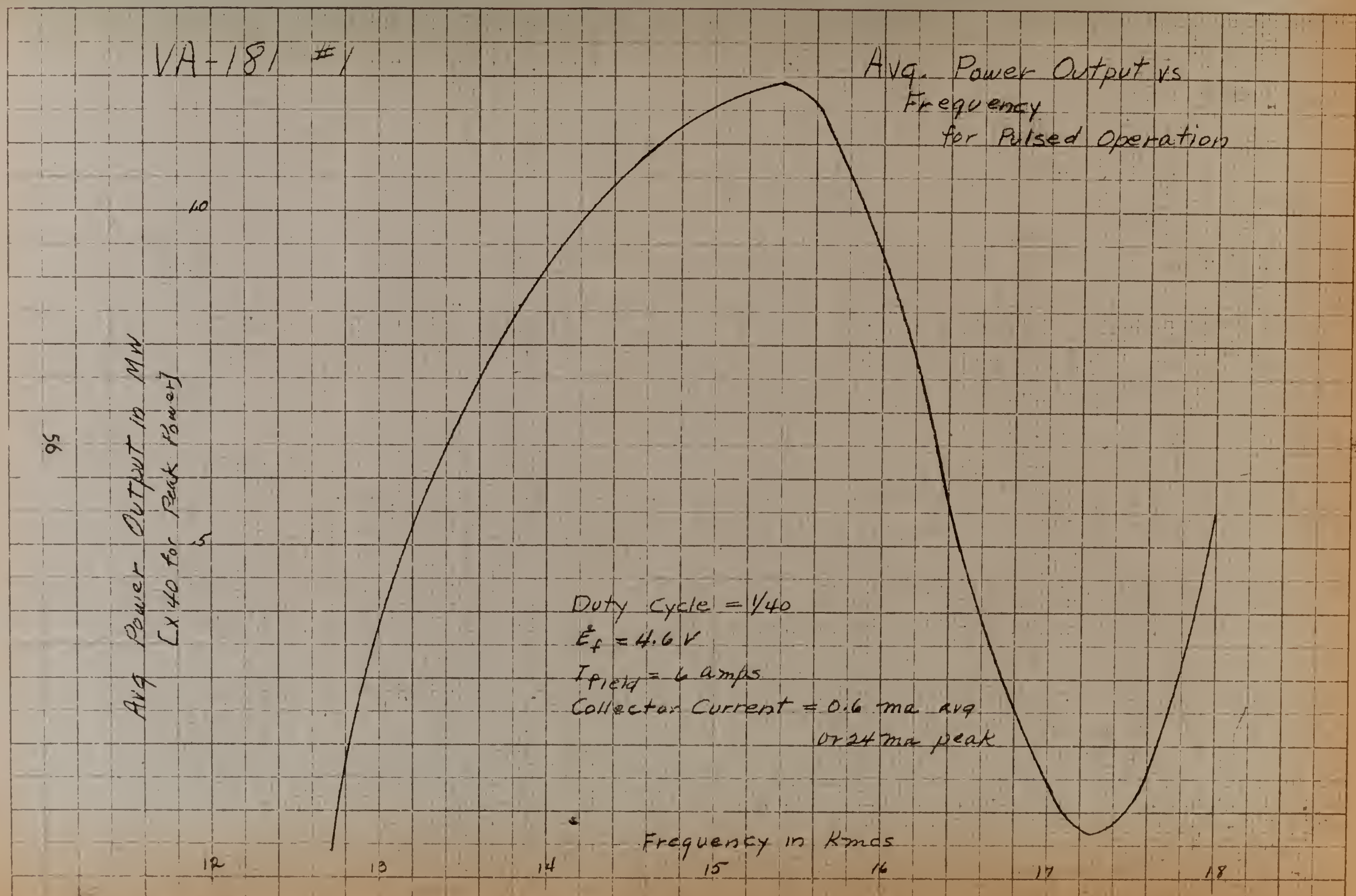

VA-181 #1
Avg. Power Output vs
Frequency
for Pulsed Operation
10
5
Avg Power Output in MW
[x 40 for Peak Power]
Duty Cycle = 1/40
$E_f$ = 4.6 V
$I_{field}$ = 6 amps
Collector Current = 0.6 ma avg
or 24 ma peak
Frequency in Kmcs
12
13
14
15
16
17
18

Fig 27

VA - 181 #1

Power Output vs Frequency
for a restricted range
CW operation

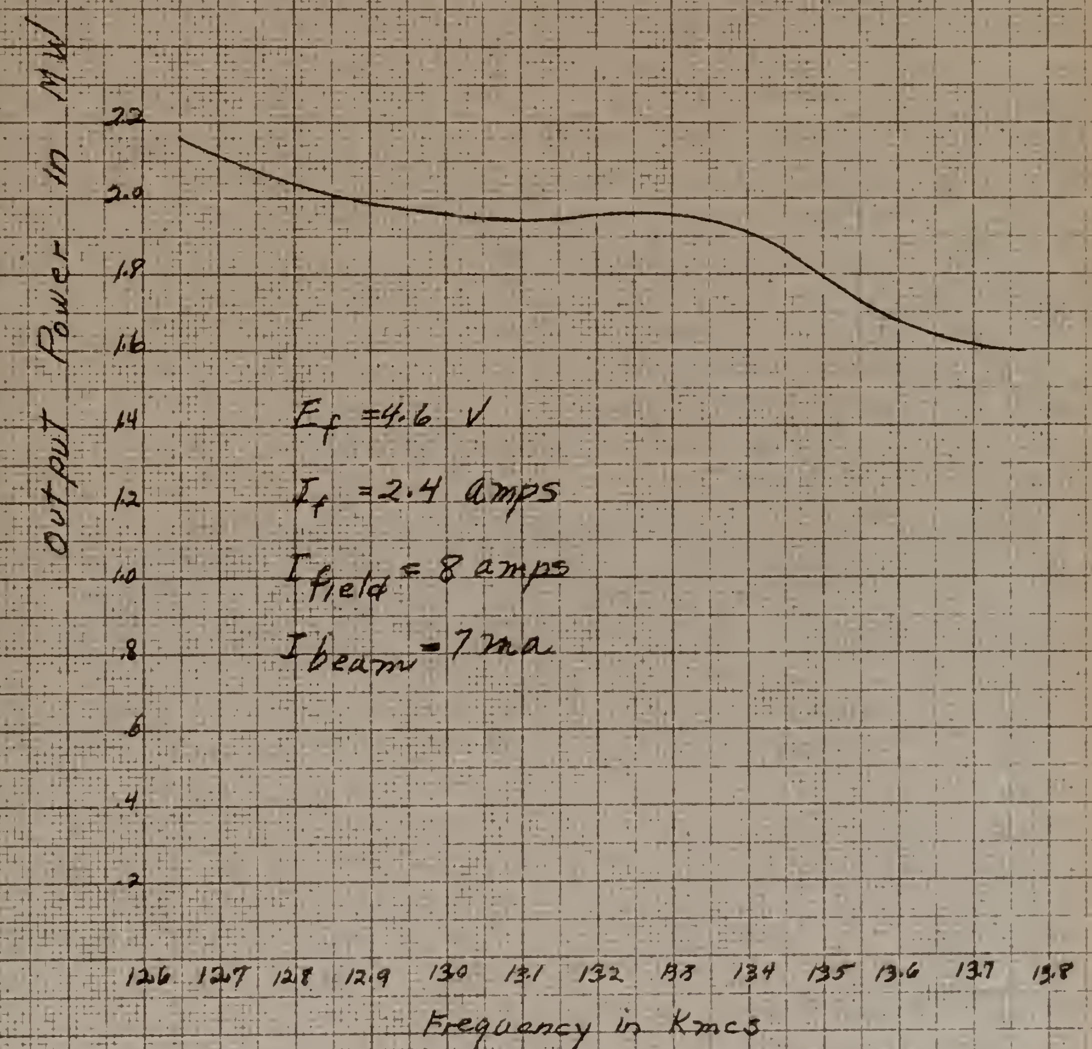



Fig 28

the variation in power output over a range of a few hundred megacycles can be made quite small (see Fig. 28 for VA-181) so that amplitude modulation may be expected to be quite low in a completed system. In the way of disadvantages of the backward wave oscillator, it, like the klystron and similar tubes, is quite frequency-sensitive to changes of supply potentials so that well regulated supplies are necessary. Also, until and unless permanent magnet focusing or electrostatic focusing of the electron beam is satisfactorily developed, the tube will require a large high-powered coil to prevent excessive interception of the electron beam by the helix or other r.f. structure. The efficiency of the backward wave oscillator is also quite low, being of the order of 1 percent or less both theoretically and actually. This figure, however, might be somewhat higher in a high powered oscillator if such can be built. The success of future developments will really be the determining factor in the practicability of the backward wave oscillator as a frequency modulated source for communication systems in the microwave range.

CHAPTER V

REACTANCE TUBE FREQUENCY MODULATION

The most direct approach to the problem of producing a frequency modulated microwave signal for communication purposes is to use the conventional or "negative grid" type tube. While it is not feasable to so modulate an oscillator of the conventional tube type operating in the kilomegacycle region, it is possible to modulate at a lower frequency and then multiply up by means of class c amplifiers to the lower kilomegacycle frequencies. The method of frequency modulating an oscillator by means of a reactance tube is well known, and is covered in most electronics textbooks. However, it will be reviewed here for completeness, and for comparison with the other devices for obtaining frequency modulated microwave signals. It should be stated here near the beginning of the chapter that this method of obtaining frequency modulation has not met with extensive use in the microwave frequency range. This lack of interest is probably the result of the difficulties encountered in multiplying up from an oscillator frequency, where reactance tube modulation is relatively easy to obtain, to an output frequency in the kilomegacycle range.

There are a large number of circuit arrangements for reactance tube modulation of an oscillator. The basic purpose, however, is to cause a tube to appear as either a capacitance or an inductance across the tuned circuit of the oscillator, and then to vary this effective capacitance or inductance by varying the grid voltage of the reactance tube with the modulating intelligence. In the most simple type of reactance tube circuits

there is little difference between that arrangement used to obtain an effective capacitance and that used to obtain an effective inductance. The analysis problem can be considered in very similar manners. Therefore only that circuitry which simulates a capacitance will be considered here. The method employed by Seeley[10] will, in general, be followed.

Referring to Fig. 29 and Fig. 30, it will be noted that the current through the series circuit of C and R is

$$I_c = \frac{E_p}{R - j\frac{1}{\omega C}}$$

and that $E_g = R I_c$ and

$$I_p = \frac{E_p + \mu E_g}{r_p} = \frac{E_p + \mu R I_c}{r_p} = \frac{E_p + \frac{\mu R E_p}{R - j\frac{1}{\omega C}}}{r_p} .$$

The total current to the reactance tube modulator is

$$I_p + I_c = E_p\left[\frac{1}{R - j\frac{1}{\omega C}} + \frac{1}{r_p} + \frac{\mu R}{r_p\left[R - j\frac{1}{\omega C}\right]}\right]$$

or

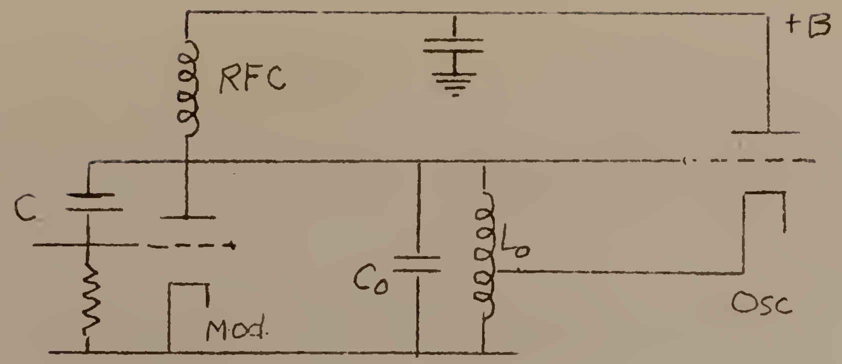
$$Y = \frac{I_p + I_c}{E_p} = \frac{1}{R - j\frac{1}{\omega C}} + \frac{1}{r_p} + \frac{1}{\frac{1}{g_m} - j\frac{1}{g_m \omega C R}}$$

which may be represented by the circuit shown in Fig. 31.

Now by proper selection of the reactance tube and the components, $\frac{1}{g_m} - j\frac{1}{g_m \omega C R}$ can be made small compared to $r_p$ and $R - j\frac{1}{\omega C}$. Further, if $\frac{1}{\omega C R}$ is large compared to 1,

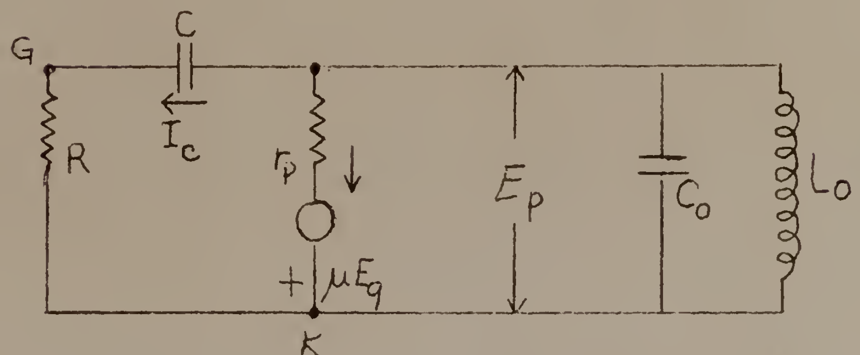
$$Y \cong -j\frac{1}{g_m \omega C R}$$

and the tube acts very nearly as a pure capacitance equal to gmCR. Further, by changing the gm of the tube, the value of this effective capacitance can be changed. The gm of a tube may be varied by varying the grid voltage. This variation may be quite linear over an extended range of grid potential (see curves on 6SG7, 6SJ7, 6SK7 and others).



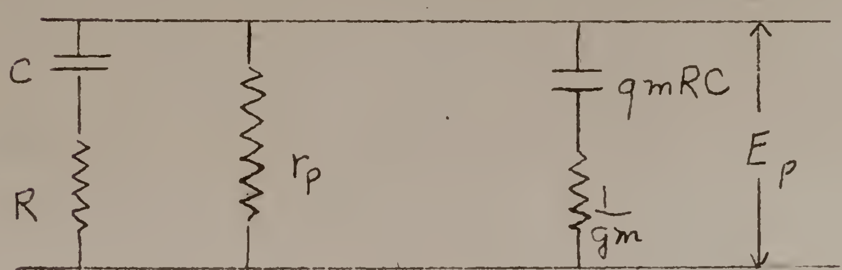

Schematic Diagram of Reactance Tube Modulator Circuit

Fig. 29



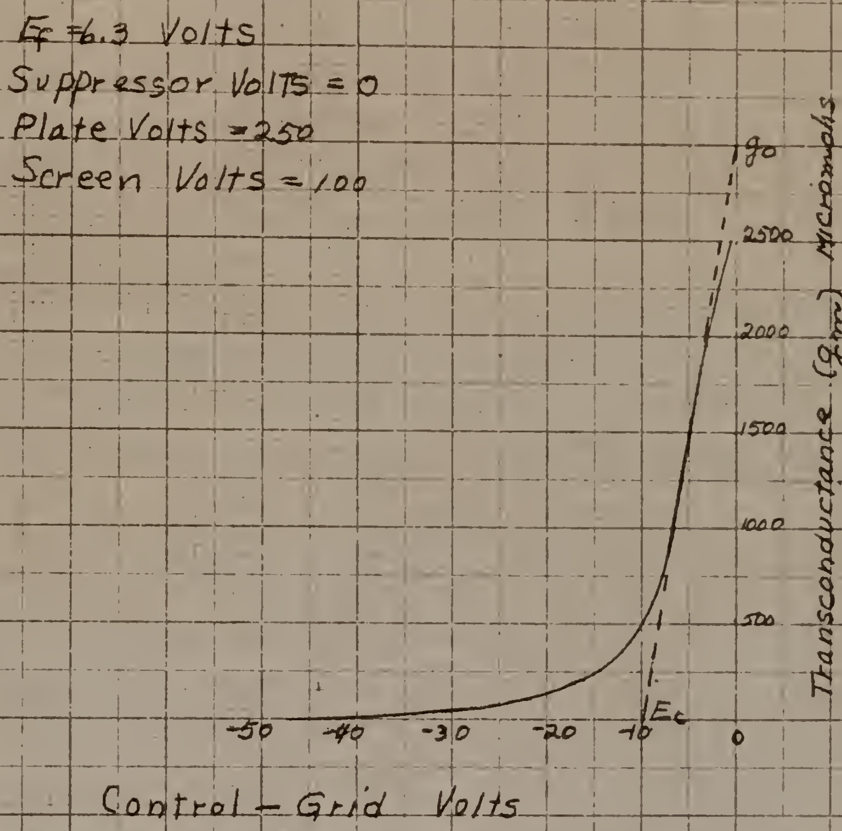

Equivalent Circuit for Reactance Tube Modulator

Fig. 30



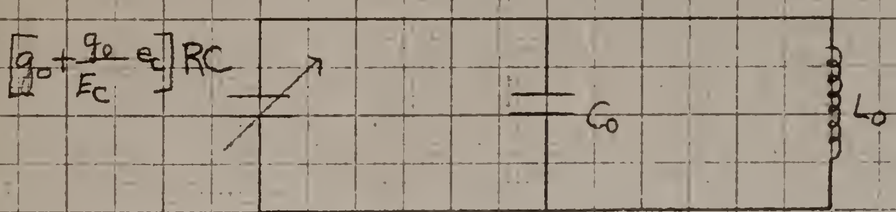

Equivalent Admittance across Oscillator Tuned Circuit

Fig. 31




Transconductance vs. Grid Voltage for 6SK7 Tube

Fig 32




Equivalent Tuned Circuit for Oscillator
Using Reactance Modulator

Fig 33

A curve for the 6SK7 is shown in Fig. 32. Over the linear range, the transconductance may be expressed as

$$g_m = g_o + \frac{g_o}{E_c} e_c$$

where $g_o$ and $E_c$ are the intercepts shown on Fig. 32 and $e_c$ is the instantaneous grid potential.

The oscillator tuned circuit may now be represented as shown in Fig. 33. The frequency may then be expressed as

$$f = \frac{1}{2\pi\sqrt{L_o\left[C_o + \left(g_o + \frac{g_o}{E_c}e_c\right)RC\right]}}$$

The modulation sensitivity

$$S = \frac{df}{de_c} = -\frac{g_o L RC}{4\pi E_c\left[L_oC_o + g_oL_oRC + \frac{g_oL_oRC}{E_c}e_c\right]^{3/2}} = \frac{-2\pi^2 g_o L_o RCf^3}{E_c}$$

and the second derivative with respect to $e_c$ is

$$\frac{d^2f}{de_c^2} = \frac{-6\pi^2 g_o L_o RCf^2}{E_c}\frac{df}{de_c} = \frac{12\pi^4 g_o^2 L_o^2 R^2C^2 f^5}{E_c^2} = \frac{-6\pi^2 g_o L_o RCf^2}{E_c} S.$$

Now in a manner similar to that used in the klystron and backward wave oscillator chapters, we again express the frequency as a Taylor expansion about v where

$$e_c = E_{cc} + v \quad \text{and} \quad v = v_m \sin \omega_m t$$

This gives

$$f(E_{cc}+v) = f(E_{cc}) + v f'(e_c)\Big|_{E_{cc}} + \frac{v^2}{2} f''(e_c)\Big|_{E_{cc}} + \cdots$$

or upon substituting

$$f(E_{cc}+v_m \sin\omega_m t) = f(E_{cc}) + v_m \sin\omega_m t \, S\Big|_{E_{cc}} - \frac{3v_m^2\pi^2 g_o L_o RC}{E_c}\sin^2\omega_m t \, f_o^2 S\Big|_{E_{cc}} + \cdots$$

Then again as $\sin^2\omega_m t = \frac{1}{2} - \frac{1}{2}\cos 2\omega_m t$,

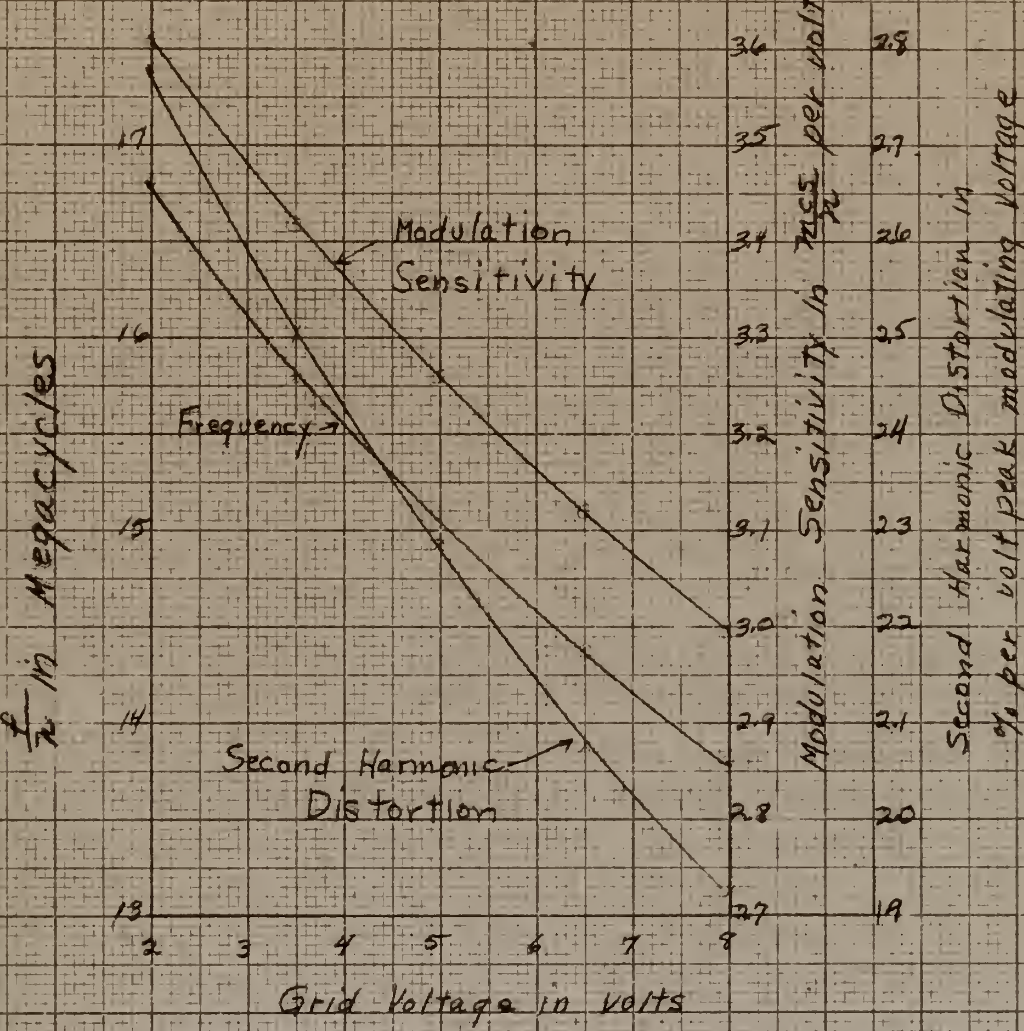
$$D_2 = \frac{3\pi^2 g_o L_o RC f_o^2 v_m}{2E_c}$$

The expression for frequency and modulation sensitivity given above are for the basic oscillator and must be multiplied by the integer, n, equal to the multiplication used to raise the signal from the oscillator frequency to the output frequency. The second harmonic distortion

expression is applicable as it is given. Fig. 34 shows curves of the frequency, modulation sensitivity, and second harmonic distortion for a hypothetical condition involving a reactance tube modulator.

As stated before, this system for obtaining frequency modulated signals at microwave frequencies does not appear to have been used to any extent. Therefore no information as to the advantages and disadvantages of this system, or any examples of performance are available. However, the normal difficulties encountered in operating negative grid tubes at kilomegacycle frequencies, even when planer tubes such as the lighthouse type are used, will be present here. The large amount of multiplication needed to obtain an output frequency sufficiently high, together with the low multiplication that can be obtained per stage, and the complexity of the circuitry of each stage, undoubtedly combine to make the operation of this kind of system impractical in view of the ease of operation of the klystron to attain the same results.

Theoretical Curves of Frequency, Modulation Sensitivity, and Second Harmonic Distortion for Reactance Tube Modulator




$L_0 = 2 \times 10^{-7}$ henries $C_o = 10^{-11}$ farads
$g_0 = 2.9 \times 10^{-3}$ mhos $E_C = 10$ volts
$R = 5 \times 10^3$ ohms $C$ $5 \times 10^{-12}$ farads
for $n=40$, $f$ goes from 1200 to 1440 mcs
Plot of theoretical Frequency, Modulation Sensitivity, and Distortion for reactance Tube Modulator

Fig 34

## CHAPTER VI

## CONCLUSION

It has not been the purpose of this paper to draw any fine comparisons between and among the various devices and methods considered beyond the short list of the more pronounced abilities and disabilities listed at the end of each chapter. The expressions showing the frequency, the modulation sensitivity, and the distortion as functions of the modulating parameter, and the various curves of these quantities are presented for any comparison that might be desired. At present there is no great competition among these devices as each seems to cover a different need. Some competition might exist in the future between the klystron and the backward wave oscillator, but even here, because of the added complexity of the backward wave oscillator, the two will probably be used in different realms with the less expensive klystron being used where the large deviations are not necessary, and the backward wave oscillator being used where advantage can be made of its ability to give large deviations.

Any of the devices or methods considered in this paper, because of their dependence upon some parameter that makes frequency modulation possible, are subject to instability of their center frequency. However they are also relatively easy to stabilize by means of automatic frequency control where the control signal may be introduced along with the normal modulating signal. This automatic frequency control would, of course, be possible even though frequency modulation was not being used, and the device may be desirable for this feature alone. The information presented here could be used, in part, for the design of circuitry for this use.

Also the output from any of the devices herein considered, if not of sufficient power in itself, may be amplified by the various methods available. Of most interest at present are the klystron amplifier and the traveling-wave tube. The traveling-wave tube is generally a broad band tube and could accomodate the output from any of these devices when designed to do so. The klystron is, however, a narrow band tube, so that the deviation would have to be small enough in order that all necessary frequency components would be passed by the klystron amplifier without extensive amplitude modulation. On the other hand, the klystron is presently capable of a much greater power output than is the traveling-wave tube.

This paper has taken the systems which are presently operational and has investigated their modulation characteristics, using first order theory. Where information of an experimental nature was available, or could be taken, such was done, and it demonstrated that in these cases, the first order theory gave reasonably good results. Also these examples demonstrated some of the capabilities of the devices at the present stage of their development. It is hoped that the equations reproduced or developed here may be of some use to the person who has the job of selecting a generator for employment in a frequency modulated system in the micro-wave region, or to the person who designs the auxiliary circuits for such employment.

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